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# **A Cartesian co-ordinate system for representing the second to fifth metacarpals in the human hand**

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## **Abstract**

### **Purpose**

The use of hand radiographs has both clinical and anthropometric applications. However, a method for converting standard bony points within the metacarpus to Cartesian co-ordinates does not exist.

### **Methods**

A simple method for converting standard bony points of the second to fifth metacarpals to Cartesian co-ordinates is described for the first time.

### **Results**

Using a small set of measurements and treating these with equations of known voracity, this method is accurate and allows the metacarpus to be interrogated via a much wider range of geometrical techniques than has so far been available.

### **Conclusions**

This method allows naked-eye assessments to be supported or replaced by metrical evaluations. It is likely to have both clinical and anthropometric uses.

## **Introduction**

Although the human hand receives attention from clinicians and anthropologists, the information obtained is usually somewhat limited. Previous work has focused on bone lengths, and sometimes widths, or the relative projection of key points to give digital or metacarpal formulae with the latter often relying more on anthroposcopy than metrical techniques.<sup>1-4</sup> Yet the morphological study of the hand is important for clinical, biomechanical, ergonomic and anthropological reasons.

Clinical radiographs are an important source of anthropometric data<sup>5</sup> including the hand.<sup>6,7</sup> In the past, the clinical use of hand radiographs has benefited from the work of anthropologists who have contributed metrical methods to aid diagnosis.<sup>8-13</sup>

Although the digits, arranged as a series of hinged long bones, appear to lend themselves to little more than simple linear measurement, the metacarpals

merit closer attention. By converting the points between which the metacarpal lengths have typically been measured<sup>14</sup> into a series of Cartesian co-ordinates, a wider range of metrical and statistical options becomes available. These include the ability to determine the distances between any pair of points, angles between any set of three points and the areas bounded by any set of three or more points.

## Methods

Standard dorsi-palmar projection radiographs of the hand are used.<sup>15, 16</sup> For purposes of this method, only the second to fifth metacarpals are considered since the mobility of the first metacarpal prevents it from adopting a reliably consistent position during radiographic examination.

For all measurements and calculations, the origin (0,0) for this co-ordinate system is chosen as the point where the centre line of the shaft of third metacarpal crosses its base. This and all the other points between which measurements are taken are the same as those already adopted.<sup>14</sup> The metacarpal heads and bases are numbered as in Fig. 1. For consistency, all the metacarpal heads are given odd numbered co-ordinates and the bases are even numbered.

Rather than simply measuring the lengths of the second to fifth metacarpals between the points set out by Parish<sup>14</sup> a form of triangulation based upon these points is adopted (Fig. 2). This triangulation allows the polar co-ordinates of each of the points at the heads and bases of the metacarpals to be determined. Polar co-ordinates take the form  $P(r, \theta)$ , where the point  $P$  is characterized by  $r$ , the distance of that point from the origin, and  $\theta$ , the angle, relative to the horizontal axis, that that point subtends at the origin. The polar co-ordinates can then be converted to Cartesian co-ordinates using standard formulae.

The necessary distances are measured directly in Step 1, the angles are calculated in Step 2 and the conversion of polar to Cartesian co-ordinates is performed in Step 3, as described below.

Only points  $(x_2, y_2)$  to  $(x_7, y_7)$  need to be considered. Since the long axis of the third metacarpal lies in the plane of the  $y$ -axis, the  $x$  co-ordinate of point  $(x_1, y_1)$  is, by definition, zero and the  $y$  coordinate is the bone's measured length.

### Step 1. Linear measurement

The following linear measurements are taken directly from the radiograph, in millimetres, using callipers accurate to 0.01 mm:

(a) The lengths of the second to fifth metacarpals (BC, AD, EF and GH in Fig.

- 2) and the distances between the:
- (b) Second and third, third and fourth and third and fifth metacarpal heads (CD, DE and DG in Fig. 2)
  - (c) Second and third, third and fourth and third and fifth metacarpal bases (AB, AF and AH in Fig. 2)
  - (d) Base of the third metacarpal and the head of the second metacarpal (AC in Fig. 2)
  - (e) Base of the third metacarpal and the head of the fourth metacarpal (AE in Fig. 2)
  - (f) Base of the third metacarpal and the head of the fifth metacarpal (AG in Fig. 2)

These lengths constitute the sides of a series of triangles (Fig. 2).

### **Step 2. Calculation of angles**

Given the lengths of the sides of the triangles constructed in Fig. 2, the internal angles may then be calculated. Using a version of the half-angle formula, six such angles are calculated. The individual equations (Eqs. (1A-1F)) used for each angle and the general form is given in Table 1. These angles and composites thereof provide the angular part of each polar co-ordinate.

Having measured the distances between the origin (0,0) and points  $(x_2, y_2)$  to  $(x_7, y_7)$  (Step 1) and calculated the angles each subtends at the origin relative to the horizontal axis (Step 2), all the elements of the polar co-ordinates of these points are known and can now be constructed (Table 2).

### **Step 3. Converting from polar to Cartesian co-ordinates**

Having constructed the polar co-ordinates of the heads and bases of second to fifth metacarpals, they may now be converted to Cartesian co-ordinates using standard formulae. The general form and the individual equations for each conversion are given in Table 3.

Usefully, having made the original measurements in millimetres, the numerical values of the calculated co-ordinates are also in millimetres.

## **Results**

### **Accuracy**

As this method turns linear measurements into Cartesian co-ordinates, tests evaluating its accuracy should seek to explore how closely the calculated co-ordinates match the relevant points on the metacarpals. Thus, to test the

closeness of these co-ordinates with the points they represent, one may plot them, full size on paper, place this on a light-table, overlay the original radiograph and align the respective points. The level of agreement between the original and the plotted points provides the degree of confidence one has in the method. When this is performed, that level of agreement has been found to be so great as to defy direct measurement.

However, it should be noted, that since this method uses mathematical calculations of known veracity, any errors that occur do not result from the calculations themselves but are those typical of the measuring processes.<sup>17</sup> Thus, provided the initial measurements are made accurately, this method is independent of further error.

This set of co-ordinates constitutes all the points that characterize the extent of the second to fifth metacarpals. Further recourse to the original radiographs is unnecessary and additional measurement error is thereby avoided.

## **Applications**

Having calculated this series of co-ordinates, it is possible to determine the distances between any two of the eight points characterized, the angles between any three of these points and the areas between any set of three or more points using standard equations in Cartesian geometry.

The angles by which the second, fourth and fifth metacarpals diverge from the central axis of the third metacarpal and their relative degree of divergence from each other, can now be determined without the need to draw projection lines on the original radiographs, as has previously been the case.

Points  $(x_2, y_2)$  to  $(x_7, y_7)$  are no longer merely points between which simple linear measurements are made but now have a precise two-dimensional spatial distribution relative to the origin- (The distribution of point  $(x_1, y_1)$  lies along the  $y$ -axis only.) Thus, population and sexual dimorphic comparisons between the spatial distribution of respective pairs of co-ordinates can now be made using statistical techniques such as Hotelling's two-sample  $T^2$  test.<sup>18</sup>

Once calculated, it is also a simple process to standardize all co-ordinates to a given metacarpal length. Standardization for size allows shape differences to be more readily appreciated statistically or by plotting wire diagrams of these metacarpals (similar to that used in Fig. 2). This is useful in morphological growth studies or when following the progress of degenerative changes such as those occurring in rheumatoid arthritis.

As presented, this method relates to the human metacarpus but, in principal, it may also be applicable to the metacarpus (and, indeed, the metatarsus) of other species. Thus, intra- and inter-species comparisons are also possible.

This method also allows metacarpal formula (the relative degree of projection of the heads of the metacarpals) to be represented by the odd numbered  $y$

co-ordinates. Where the phalangeal lengths for each finger are also known, these may be added to these y co-ordinates to give a metrical version of digital formula (the relative degree of projection of the tips of the fingers). As this may not include the width of the joint spaces or other soft tissues, this does not correspond to the sizes to be expected in a live hand.

## **Conclusions**

It is already known that a number of genetic syndromes are associated with alteration in skeletal geometry of the hand. For example, shortening of the fourth metacarpal (brachymetacarpia-4), also known as Metacarpal or Archibald's Sign, which occurs in a number of clinical and sub-clinical genetic conditions.<sup>19</sup> Frequently, assessment of such cases relies on visual judgement. With a more sensitive metrical tool it may be possible to refine such judgements. Using a few simple measurements, quickly and easily entered into a computer spreadsheet application where the calculations can be performed rapidly and reliably, a more detailed impression may be obtained.

This method converts simple linear measurements, taken without the need for specific orientation of the original radiograph, to a system of orthogonal co-ordinates so that the region between the second to fifth metacarpals can be subjected to a greater range of analysis using Cartesian geometric and statistical techniques. It is a low cost, relatively simple, yet effective method. Using proven mathematical formulae, this method is accurate and trustworthy. It allows the collection of a body of data that more accurately depicts the metacarpus by introducing a spatial distribution where previously only individual lengths were taken and their separation ignored. The co-ordinate database generated can be readily re-worked, as new ideas arise, without having to take new measurements or resorting to the original radiographs (should they still be available) and the introduction of further error avoided.

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Figure 1 – Outline of the metacarpus showing co-ordinate points

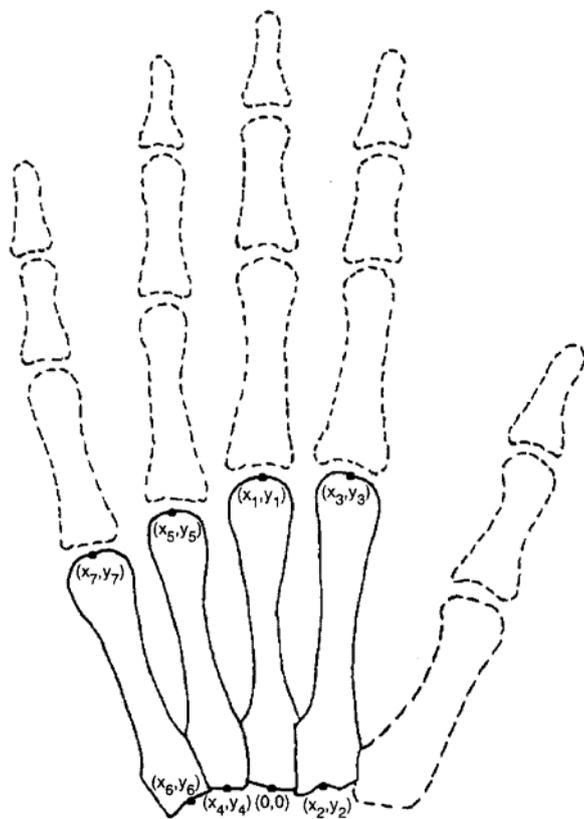
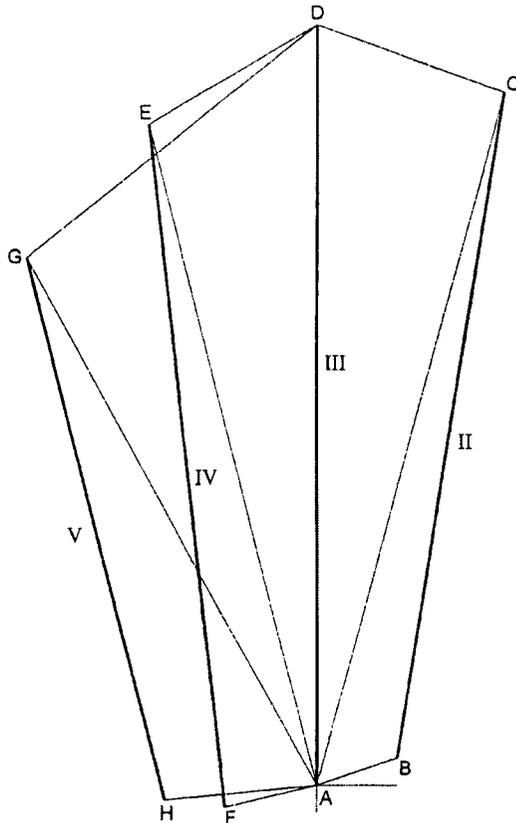


Figure 2 – Triangulation of the metacarpus



Heavy lines, with Roman numerals, indicate the metacarpals (the lengths of which are measured); light lines indicate other measured distances between the metacarpal heads and cases. Step 1, distances, BC, AD, EF, GH; CD, DE, DG; AB, AF, AH; AC, AE and AG are measured. In Step 2, angles:

$\widehat{CAD}$ ,  $\widehat{DAE}$ ,  $\widehat{EAF}$ ,  $\widehat{DAG}$ ,  $\widehat{DAH}$  are calculated.

**Table 1 – Formulae for calculating required angels**

General form

All angles are determined using the sine version of the 'half-angle formula' for angle  $A$  in triangle  $ABC$ :

$$\sin \frac{A}{2} = \sqrt{\frac{(S-b)(S-c)}{bc}}$$

re-arranged into the inverse sine form of the same equation:

$$A = 2 \times \sin^{-1} \sqrt{\frac{(S-b)(S-c)}{bc}}$$

where  $S$  is the semi-perimeter of the triangles within which the angle appears, and  $b$  and  $c$  are the sides adjacent to the angle ( $A$ ) being calculated.

Specific formulae

$$\widehat{BAC} = 2 \times \sin^{-1} \sqrt{\frac{(S_{ABC} - AB)(S_{ABC} - AC)}{(AB \times AC)}} \quad (1A)$$

$$\widehat{CAD} = 2 \times \sin^{-1} \sqrt{\frac{(S_{ACD} - AC)(S_{ACD} - AD)}{(AC \times AD)}} \quad (1B)$$

$$\widehat{DAE} = 2 \times \sin^{-1} \sqrt{\frac{(S_{ADE} - AD)(S_{ADE} - AE)}{(AD \times AE)}} \quad (1C)$$

$$\widehat{EAF} = 2 \times \sin^{-1} \sqrt{\frac{(S_{AEF} - AE)(S_{AEF} - AF)}{(AE \times AF)}} \quad (1D)$$

$$\widehat{DAG} = 2 \times \sin^{-1} \sqrt{\frac{(S_{ADG} - AD)(S_{ADG} - AG)}{(AD \times AG)}} \quad (1E)$$

$$\widehat{DAH} = 2 \times \sin^{-1} \sqrt{\frac{(S_{DAH} - AD)(S_{DAH} - AH)}{(AD \times AH)}} \quad (1F)$$

**Table 2 – Polar co-ordinates, in the form  $p(r,\theta)$ , of each of the Cartesian co-ordinates**

$$(x_0, y_0) = (U, U)$$

$$(x_1, y_1) = (AD, 0)$$

$$(x_2, y_2) = (AB, (90^\circ - (\widehat{BAC} + \widehat{CAD}))) \text{ nb } (\widehat{BAC} + \widehat{CAD}) \equiv \widehat{BAD}$$

$$(x_3, y_3) = (AC, (90^\circ - \widehat{CAD}))$$

$$(x_4, y_4) = (AF, (90^\circ + (\widehat{DAE} + \widehat{EAF}))) \text{ nb } (\widehat{DAE} + \widehat{EAF}) \equiv \widehat{DAF}$$

$$(x_5, y_5) = (AE, (90^\circ + \widehat{DAE}))$$

$$(x_6, y_6) = (AH, (90^\circ + (\widehat{DAG} + \widehat{GAH}))) \text{ nb } (\widehat{DAG} + \widehat{GAH}) \equiv \widehat{DAH}$$

$$(x_7, y_7) = (AG, (90^\circ + \widehat{DAG}))$$

**Table 3 – Formulae for converting polar to Cartesian co-ordinates**

General form

$$x = r \cos \theta \quad (2a)$$

$$y = r \sin \theta \quad (2b)$$

where  $r$  is the measured distance between the base of the metacarpal III and the head or base of each metacarpal and  $\theta$  is the angle subtended by that line to the horizontal axis (as calculated in Step 2).

Specific form

The equations used to determine co-ordinates  $(x_1, y_1)$  to  $(x_7, y_7)$ .

$$\begin{cases} x_1 = 0 \\ y_1 = AD \end{cases}$$

$$\begin{cases} x_2 = AB \cos(90^\circ - (\widehat{BAC} + \widehat{CAD})) \\ y_2 = AB \sin(90^\circ - (\widehat{BAC} + \widehat{CAD})) \end{cases}$$

$$\begin{cases} x_3 = AC \cos(90^\circ - \widehat{CAD}) \\ y_3 = AC \sin(90^\circ - \widehat{CAD}) \end{cases}$$

$$\begin{cases} x_4 = AF \cos(90^\circ + (\widehat{DAE} + \widehat{EAF})) \\ y_4 = AF \sin(90^\circ + (\widehat{DAE} + \widehat{EAF})) \end{cases}$$

$$\begin{cases} x_5 = AE \cos(90^\circ + \widehat{DAE}) \\ y_5 = AE \sin(90^\circ + \widehat{DAE}) \end{cases}$$

$$\begin{cases} x_6 = AH \cos(90^\circ + (\widehat{DAG} + \widehat{GAH})) \\ y_6 = AH \sin(90^\circ + (\widehat{DAG} + \widehat{GAH})) \end{cases}$$

$$\begin{cases} x_7 = AG \cos(90^\circ + \widehat{DAG}) \\ y_7 = AG \sin(90^\circ + \widehat{DAG}) \end{cases}$$