Integrated Design of Optimisation and Inverse Dynamics Control for Home Heating Systems*

C. P. D. Downing and J. M. Counsell

Abstract— **This paper presents a novel controller design methodology for multi-input multi-output linear time invariant systems using a realisable system's inverse dynamics whilst simultaneously aiming to optimize the systems performance. This is very useful for energy system type controller design problems, especially heating systems as described in this paper. The new methodology, OPTimal Inverse Control (OPTIC) integrates the challenge to minimize a cost function and introduce it into the feedback control solution, thus allowing for simultaneous fast acting stable and non-interacting feedback control with cost function minimisation to provide a robust and optimal solution. The cost function presented acts to minimise weighted proportions of Carbon Intensity (c_I) of the energy supply system and the Cost of the Tariff (c_{Tar}) depending on the policy weighing potential fuel poverty against net zero carbon heating solutions. It is described how the OPTIC method is applied to a heating system control problem using a home heating dynamic model Inverse Dynamics based Energy Assessment and Simulation (IDEAS). IDEAS calibrated with UK's Part L building regulations models a home's building physics, heating demand using a standard type of occupancy over a whole year. This allows the OPTIC method to control the heating system to reach the required home comfort whilst continuously heading towards the minimum cost function value at all times, minute by minute instead of a much slower period of optimisation that is achieved by other complementary methods, like General Predictive Control (GPC).**

I. Introduction

With the rise for the need for carbon friendly heating solutions in the UK (and the world), there is a subsequential rise in the need for accurate adaptable models that can model modern technologies efficiently with fast simulation speeds and high precision. One such model named Inverse Dynamics based Energy Assessment and Simulation (IDEAS) [7] seeks to do this with its calibrated methodology against the Standard Assessment Procedure (SAP), developed by the Building Research Establishment (BRE) in the UK. SAP is the UK's Part L building regulation methodology used to assess a home's energy requirements to meet the thermal comfort requirements for the space and to show a home's compliance with the building regulations [4]. First published in 1993, the SAP methodology has gone through a series of iterative updates over the years leading to the current documentation, SAP 10.2 [1]. The SAP calculation methods for energy, cost and carbon use the BRE Domestic Energy Model (BREDEM) with the latest BREDEM revision being BREDEM 12 [5]. The SAP documentation and calculation methodologies are also used to create an Energy Performance Certificate (EPC), which is a government requirement when building, selling, or renting a property within the UK. This certificate aims to ensure the building is in alignment with the specifications of a building laid out by the government [3].

The IDEAS methodology is an Inverse Dynamics based model that was first introduced as IDEAS in 2011 [6]. IDEAS was derived from the previously published building physics models shown in [2]. The model in [7] is a single zone, small perturbation model that uses four temperature nodes to depict the state variables within the state space model which are:

- Zone Temperature, T_{zone} representing the temperature of the air and of the furniture.
- Structural Temperatures, $\overline{T_{sl}}$ and $\overline{T_{se}}$ for the internal structural wall and external structural wall respectively and taken from the centre point of each construction.
- Internal Mass Temperature, T_{im}. This is representative of the non-removable mass within a building.

Alongside the requirement for accurate models, there is a need for the models to be optimised sufficiently to produce the best results for a given system. One of the biggest issues found with optimisation models is that they have a substantial penalty on the speed of a systems simulation, caused by the model needing to be run multiple times to find an optimum solution. This additional time penalty of optimisers in a system can lead to system instabilities when integrated in the feedback control, or reduced productivity when used in design processes. In this paper the challenge of feedback control is addressed and there is a need for a new optimisation methodology that minimises the time deficit in simulation caused by the optimisation algorithm.

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This is especially important in complex optimisation strategies that look to not only optimise a single value but optimise an array of values within a system.

The key aim for this research is to develop a new optimisation algorithm to be integrated the into the building physics Inverse Dynamics (ID) model to allow for fast feedback control with integrated optimisation to add robustness and error correction to the system. The cost functions will act to minimise weighted proportions of Carbon Intensity (C_I) and Cost Tariff (C_{Tar}) with the weights dependent on policy of striving for net zero carbon heating solutions against fuel poverty. When implemented the reference set point for the cost function will be set to zero to ensure the system is always aiming towards a minimum C_I and C_{Tar}. This work will be building off the foundation blocks of the optimisation models currently being used. This new theory and methodology will be named OPTimal Inverse Control (OPTIC). The OPTIC methodology will be embedded within the inverse dynamics state space methodology used by the IDEAS methodology allowing the system to always perform stable and accurate thermal comfort whilst maintaining the objective/ cost function. This will be inherently different to the other optimisation models as other models are wrap around based, i.e. slow as opposed to fast acting.

II. Methodology

The research into the OPTIC method will comprise of three stages that build up the model in complexity comparing the results against a baseline set of results from both the SAP method and the IDEAS results. These stages are described as follows:

- Stage 1- Simple model consisting of the IDEAS state space with the addition of a basic battery with a cost function as a measured additional output.
- Stage 2- This stage will develop the model to include a simple battery within the output vector along with a more complex cost function made up of components to minimise the carbon and cost of the system.
- Stage 3- The final stage for OPTIC development will be to enhance the model of stage 2 to include a singular perturbation model for the cost function which will generate an addition of input and output disturbance vectors in the state space model.

Each of the stages will also be compared against themselves to determine whether they are having not only a positive impact, but also aren't at the detriment of the feedback control system's stability. An optimal and stable solution will allow the building physics part of the model to still maintain control. The optimisation algorithm will be deemed unsatisfactory if it tampers with the feedback control system's ability to maintain control of the building dynamics and the accurate control of the occupants' thermal comfort. To assess the merits of each of the stages, a coefficient of determination algorithm will be used comparing solely against the model's calibrated data provided by the SAP calculation methodology.

III. OPTIC Theory

The OPTIC theory will comprise of the three stages discussed within the methodology to build up the complexity to allow for analysis at each stage. All three stages will use a stripped back ID control law of the RIDE control law used in [7].

A. Stage 1

The first of three stages will be to include a simple battery component within the state space building physics model so that the building heating demand is provided by the battery (currently with no limits). This stage will also have the cost function as an addition to the output vector.

The rate of change regarding the battery ($\dot{x}_{soc}(t)$) can be expressed as the charge rate of the battery minus the discharge rate of the battery, shown in (1).

$$\dot{x}_{soc}(t) = v(t) - u(t). \tag{1}$$

where $v(t)$ is the charge rate and $u(t)$ is the discharge rate. The discharge rate of the battery can be expressed as (2) below which allows for the transformation of $u(t)$ into $u_{comp}(t)$ for the future modelling of a heat pump within the system.

$$u(t) = \mathrm{COP}_0 u_{comp}(t). \tag{2}$$

where COP_0 is the initial Coefficient of Performance of the heater, which in this case will be set to 1 for the purpose of modelling a Direct Electric (DE) acting system. This therefore makes

$$\dot{x}_{soc}(t) = v(t) - \mathrm{COP}_0 u_{comp}(t). \tag{3}$$

The cost function selected for this stage surrounds the battery and minimising the state of charge of the battery and can be expressed as (4) where ${}^{\Gamma}t$ is the period of optimisation.

$$\Gamma(t) = \int_0^{\Gamma_t} (u(t)^2 + v(t)^2)dt\,. \tag{4}$$

For use in the state space model, the cost function will be the derivative of (4) and then using singular perturbation theory, the cost function can be expressed as (5) and then as (6) to involve the compressor component.

$$\dot{\Gamma}(t) = 2\mathrm{u}_0 u(t) + 2\mathrm{v}_0 v(t). \tag{5}$$

$$\dot{\Gamma}(t) = 2\mathrm{u}_0 \mathrm{COP}_0 u_{comp}(t) + 2\mathrm{v}_0 v(t). \tag{6}$$

where u_0 and v_0 are the initial discharge and charge rates respectively for the battery which will both be set to one to induce an initial steady state position within the battery.

To remove any confusion between the building physics model state space components and the new OPTIC theory components, the OPTIC components will be pre superscripted with the Γ notation. For example, the input vector in OPTIC will be referred to as ${}^{\Gamma}u(t)$. The OPTIC state space equations are defined in (7) and (8).

$${}^{\Gamma}\dot{x}(t) = {}^{\Gamma}\mathrm{A}\,{}^{\Gamma}x(t) + {}^{\Gamma}\mathrm{B}\,{}^{\Gamma}u(t) + {}^{\Gamma}\mathrm{E}\,{}^{\Gamma}d(t). \tag{7}$$

$${}^{\Gamma}y(t) = {}^{\Gamma}\mathrm{C}\,{}^{\Gamma}x(t) + {}^{\Gamma}\mathrm{D}\,{}^{\Gamma}u(t) + {}^{\Gamma}\mathrm{F}\,{}^{\Gamma}d(t). \tag{8}$$

where ${}^{\Gamma}\dot{x}(t)$ defines the state equation, ${}^{\Gamma}x(t)$ is the state variables vector comprising of $\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{soc}(t) \end{bmatrix}$, ${}^{\Gamma}y(t)$ is the output

vector comprising of $[y(t) \quad \dot{\Gamma}(t)]^T$ and ${}^{\Gamma}d(t)$ is the disturbance vector. However, in this stage, there will be no disturbances which renders ${}^{\Gamma}\mathbf{E} = {}^{\Gamma}\mathbf{F} = 0$. ${}^{\Gamma}\mathbf{A}$, ${}^{\Gamma}\mathbf{B}$, ${}^{\Gamma}\mathbf{C}$ and ${}^{\Gamma}\mathbf{D}$ are all the system matrices. The final state and output equations can now be formed to produce (9) and (10).

$$
\begin{aligned}
{}^{\Gamma}\dot{\mathrm{x}}(\mathrm{t}) &= \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ x_{soc}(t) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{BCOP_0} & \mathbf{0} \\ -\mathbf{COP_0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v(t) \end{bmatrix}. \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
{}^{\Gamma}\mathrm{y}(\mathrm{t}) &= \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ x_{soc}(t) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{DCOP_0} & \mathbf{0} \\ \mathbf{2U_0COP_0} & \mathbf{2V_0} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v(t) \end{bmatrix}. \qquad (10)
\end{aligned}
$$

where A, B, C and D are the system matrices of the building physics model defined in [7].

This system has been simulated in MATLAB and Simulink to track a constant reference signal of 21℃ and 0 for the cost function (to ensure it is always striving towards the minimum) respectively over a 1-day period. The results of this can be seen in Fig. (1-3) representing the temperature output, cost function output and cost function respectively.

As can be seen in the figures, the addition of the OPTIC stage 1 methodology showed to have no effect on the heating controllability as Fig. (1) demonstrates the system increasing to 21℃ and then remaining there with no steady state error. The gamma derivative output, Fig. (2) acts as expected in relation to the temperature with an instant decrease and then a slow stabilisation as the temperature demand is met. It doesn't however, manage to meet the reference input of 0, but this error can be associated with the cost function minimising as much as it can whilst maintaining the temperature demand of the zone. As seen towards the end of the simulation period, the cost function is stabilising at a point that is its minimum whilst maintaining the temperature set point. Finally, the cost function, Fig. (3), has a predicted response slowly rising over the period due to it being the integral of the square of the charge and discharge rates summed together.

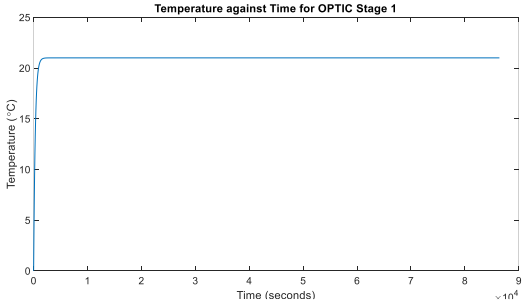

Figure 1-Temperature (Degrees Celsius) against Time (Seconds) for OPTIC Stage 1.

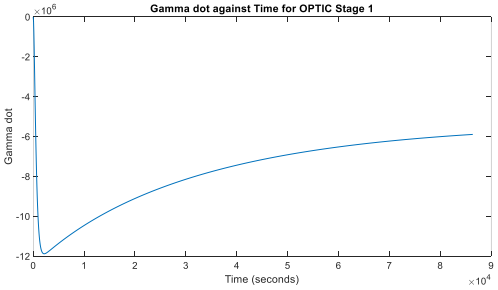

Figure 2- Γ̇against Time (Seconds) for OPTIC Stage 1.

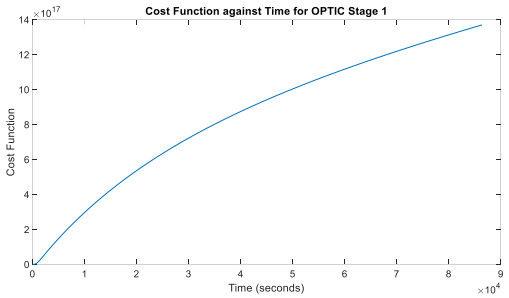

Figure 3- Cost Function against Time (Seconds) for OPTIC Stage 1.

B. *Stage 2*

In the second OPTIC stage, the system is developed to have the output vector comprise of the temperature set point ($y_T(t)$), state of charge of the battery ($y_{soc}(t)$) and the derivative of the cost function ($\delta\dot{\Gamma}(t)$) which is defined as (11). This provides an advantage over the work of stage 1 as it allows the battery to be controlled.

$$
{}^{\Gamma}\mathrm{y}(\mathrm{t}) = [y_T(t) \quad y_{soc}(t) \quad \delta\dot{\Gamma}(t)]^{\mathrm{T}}. \qquad (11)
$$

The cost function has undergone an update to reflect the additional complexity of stage two. The new cost function, (12) and (13), has been developed to include a minimisation function for the Carbon Intensity (C_I) and Cost Tariff (C_{Tar}) against the electrical grid's energy supply to the battery and to the heating system. At this stage however, the C_I and C_{Tar} values are being treated as constants.

$$
\begin{aligned}
\Gamma(t) &= \mathrm{W_1 C_I} \int_0^{\Gamma_t} (v_1(t)^2 + v_2(t)^2)dt + \\
&\mathrm{W_2 C_{Tar}} \int_0^{\Gamma_t} (v_1(t)^2 + v_2(t)^2)dt. \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\delta\dot{\Gamma}(t) &= 2\mathrm{W_1 C_I}(\mathrm{v_1}(0)\delta v_1(t) + \mathrm{v_2}(0)\delta v_2(t)) \\
&+2\mathrm{W_2 C_{Tar}}(\mathrm{v_1}(0)\delta v_1(t) + \mathrm{v_2}(0)\delta v_2(t)). \qquad (13)
\end{aligned}
$$

where W_1 and W_2 are the weights and v_1 and v_2 are the grid powers into the battery and general system respectively. $\mathrm{v_1}(0) = \mathrm{v_2}(0) = 1$.

To ensure that the system remains square, allowing it to be successfully inverted, the state vector and input vector have also been expanded, (14) and (15).

$$
{}^{\Gamma}x(t) = \begin{bmatrix} x_T(t) \\ x_{soc}(t) \\ \delta\Gamma(t) \end{bmatrix} .. (14) \quad {}^{\Gamma}u(t) = \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix} .. (15)
$$

Where $u_B(t)$ is the battery output.

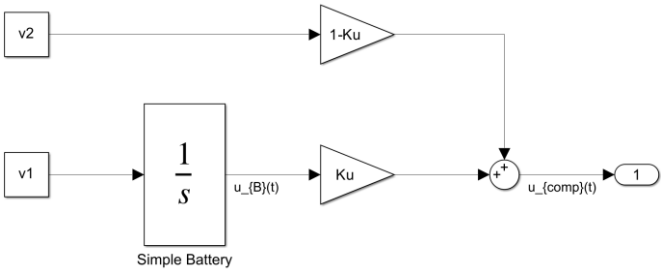

Figure 4- Simple Battery Layout block diagram.

Fig. (4) represents the simple battery system where K_u is a set value between 0 and 1 to vary the amount of power supplied from the battery and supplied from the grid directly. From Fig. (4), (16) can be derived.

$$v_2 = \frac{1}{1-\mathrm{K_u}}u_{comp}(t) - \frac{\mathrm{K_u}}{1-\mathrm{K_u}}u_B(t). \tag{16}$$

Therefore, the cost function can be updated to (17).

$$\delta\dot{\Gamma}(t) = \mathrm{B_\Gamma}v_1(t) + \frac{\mathrm{B_\Gamma}}{1-\mathrm{K_u}}u_{comp}(t) - \frac{\mathrm{B_\Gamma K_u}}{1-\mathrm{K_u}}u_B(t). \tag{17}$$

where

$$\mathrm{B_\Gamma} = 2\mathrm{W_1C_I} + 2\mathrm{W_2C_{Tar}}. \tag{18}$$

All the above can be brought together to form the final state and output equations represented by (19) and (20).

$$\begin{bmatrix} \dot{x}_T(t) \\ \dot{x}_{soc}(t) \\ \delta\dot{\Gamma}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} & \mathbf{A_{13}} & \mathbf{A_{14}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{21}} & \mathbf{A_{22}} & \mathbf{A_{23}} & \mathbf{A_{24}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{31}} & \mathbf{A_{32}} & \mathbf{A_{33}} & \mathbf{A_{34}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{41}} & \mathbf{A_{42}} & \mathbf{A_{43}} & \mathbf{A_{44}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_T(t) \\ x_{soc}(t) \\ \delta\Gamma(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B_{11}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{21}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{31}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{41}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} \\ \dfrac{\mathbf{B_\Gamma}}{\mathbf{1-K_u}} & \mathbf{B_\Gamma} & -\dfrac{\mathbf{B_\Gamma K_u}}{\mathbf{1-K_u}} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix}. \tag{19}$$

$$\begin{bmatrix} y_T(t) \\ y_{soc}(t) \\ \boldsymbol{\delta\dot{\Gamma}(t)} \end{bmatrix} = \begin{bmatrix} \mathbf{C_{11}} & \mathbf{C_{12}} & \mathbf{C_{13}} & \mathbf{C_{14}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_T(t) \\ x_{soc}(t) \\ \delta\Gamma(t) \end{bmatrix} + \begin{bmatrix} \mathbf{D_{11}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \dfrac{\mathbf{B_\Gamma}}{\mathbf{1-K_u}} & \mathbf{B_\Gamma} & -\dfrac{\mathbf{B_\Gamma K_u}}{\mathbf{1-K_u}} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix}. \tag{20}$$

where A_{i-j}, B_{i-1}, C_{1-j}, $1 < i,j < 4$ and $D_{11}$are the system matrices of the original IDEAS methodology.

The stage 2 system produced Fig. (5-8) when simulated in MATLAB and Simulink. For the simulation the set points were as follows; Temperature- 21℃, State of Charge- 3.6×10^6J = 1kWh, cost function- 0. The ratio of battery to grid into the system was set at $K_u = 0.85$.

The temperature control of the building physics model is maintained such that the temperature rises to 21℃ and then remains constant with no error. The state of charge of the battery can successfully rise to a maintain a charge of 1kWh. The cost function rate can achieve its set point of 0, albeit with extremely minor ($\times 10^{-10}$) fluctuations around to 0 point. Finally, the cost function shows an initial high altitude change and then settles which is reflective of what happens with the temperature control at the start of the simulation and then finding its steady state.

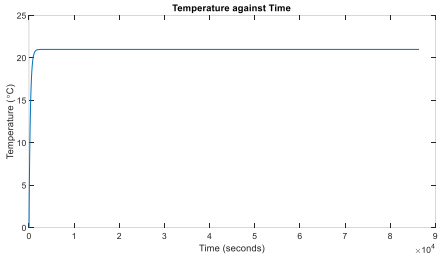

Figure 5- Temperature (Degrees Celsius) again Time (Seconds) for OPTIC Stage 2.

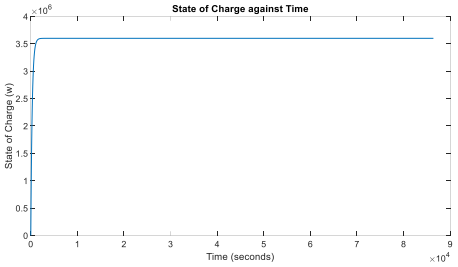

Figure 6- State of Charge (Joules) against Time (Seconds) for OPTIC Stage 2.

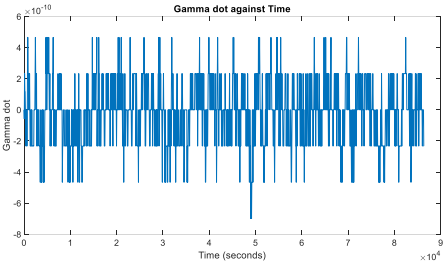

Figure 7- $\dot{\Gamma}$ against Time (Seconds) for OPTIC Stage 2.

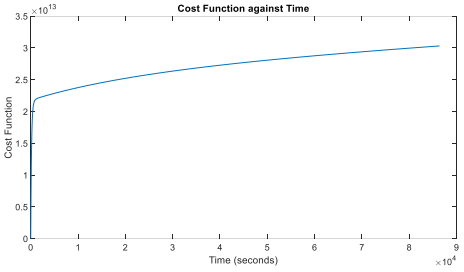

Figure 8- Cost Function against Time (Seconds) for OPTIC Stage 2.

C. *Stage 3*

The final stage of the OPTIC development isn't too dissimilar to the work of stage two with the differences coming in the form of

the simulation being for a full year running the full IDEAS model including disturbances. The cost function derivative from stage two will also undergo partial differentiation to produce a singular perturbation cost function form to be simulated. This will add carbon intensity and cost tariff components into the disturbances of the system.

With the singular perturbation theory applied to the cost function from stage 2 but now with nonconstant C_I and C_{Tar} values, (12) and (13) can be updated to (21) and (22).

$$\Gamma(t) = \mathrm{W}_1 \int_0^{\Gamma_t} c_I(t)(v_1(t)^2 + v_2(t)^2)dt + \mathrm{W}_2 \int_0^{\Gamma_t} c_{Tar}(t)(v_1(t)^2 + v_2(t)^2)dt. \quad (21)$$

$$\delta\dot{\Gamma}(t) = \mathrm{B}_{\Gamma 1}\delta v_1(t) + \mathrm{B}_{\Gamma 1}\delta v_2(t) + \mathrm{B}_{\Gamma 2}\mathrm{W}_1\delta c_I(t) + \mathrm{B}_{\Gamma 2}\mathrm{W}_2\delta c_{Tar}(t). \quad (22)$$

where

$$\mathrm{B}_{\Gamma 1} = 2\mathrm{W}_1\mathrm{c}_\mathrm{I}(0) + 2\mathrm{W}_2\mathrm{c}_{\mathrm{Tar}}(0). \quad (23)$$

and

$$\mathrm{B}_{\Gamma 2} = 2\mathrm{v}_1(0) + 2\mathrm{v}_2(0). \quad (24)$$

The cost function can undergo the same transformation as stage 2 using (16) to generate (25).

$$\delta\dot{\Gamma}(t) = \mathrm{B}_{\Gamma 1}\delta v_1(t) + \frac{\mathrm{B}_{\Gamma 1}}{1-\mathrm{K_u}}u_{comp}(t) - \frac{\mathrm{B}_{\Gamma 1}\mathrm{K_u}}{1-\mathrm{K_u}}u_B(t) + \mathrm{B}_{\Gamma 2}\mathrm{W}_1\delta c_I(t) + \mathrm{B}_{\Gamma 2}\mathrm{W}_2\delta c_{Tar}(t). \quad (25)$$

The general state space equations for this stage are expressed as (26) and (27).

$${}^{\Gamma}\dot{x}(t) = {}^{\Gamma}\mathrm{A}^{\Gamma}x(t) + {}^{\Gamma}\mathrm{B}^{\Gamma}u(t) + {}^{\Gamma}\mathrm{E}^{\Gamma}d(t). \quad (26)$$

$${}^{\Gamma}y(t) = {}^{\Gamma}\mathrm{C}^{\Gamma}x(t) + {}^{\Gamma}\mathrm{D}^{\Gamma}u(t) + {}^{\Gamma}\mathrm{F}^{\Gamma}d(t). \quad (27)$$

where

$${}^{\Gamma}\dot{x}(t) = \begin{bmatrix} \dot{x}_T(t) \\ \dot{x}_{soc}(t) \\ \delta\dot{\Gamma}(t) \end{bmatrix} \quad (28) \qquad {}^{\Gamma}x(t) = \begin{bmatrix} x_T(t) \\ x_{soc}(t) \\ \Gamma(t) \end{bmatrix} \quad (29) \qquad {}^{\Gamma}u(t) = \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix} \quad (30)$$

$${}^{\Gamma}y(t) = [y_T(t) \quad y_{soc}(t) \quad \delta\dot{\Gamma}(t)]^T. \quad (31)$$

$${}^{\Gamma}d(t) = [\delta T_0(t) \quad \delta\emptyset_{free}(t) \quad \delta c_I(t) \quad \delta c_{Tar}(t)]^T. \quad (32)$$

With all of these prerequisites, the final state space equations can be formulated as (33) and (34).

$$\begin{bmatrix} \dot{T}_{zone}(t) \\ \dot{\overline{T_{sl}}}(t) \\ \dot{\overline{T_{se}}}(t) \\ \dot{T}_{im}(t) \\ \dot{x}_{soc}(t) \\ \delta\dot{\Gamma}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A_{11}} & \mathbf{A_{12}} & \mathbf{A_{13}} & \mathbf{A_{14}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{21}} & \mathbf{A_{22}} & \mathbf{A_{23}} & \mathbf{A_{24}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{31}} & \mathbf{A_{32}} & \mathbf{A_{33}} & \mathbf{A_{34}} & \mathbf{0} & \mathbf{0} \\ \mathbf{A_{41}} & \mathbf{A_{42}} & \mathbf{A_{43}} & \mathbf{A_{44}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} T_{zone}(t) \\ \overline{T_{sl}}(t) \\ \overline{T_{se}}(t) \\ T_{im}(t) \\ x_{soc}(t) \\ \Gamma(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B_{11}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{21}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{31}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B_{41}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} \\ \dfrac{\mathbf{B_{\Gamma 1}}}{\mathbf{1-K_u}} & \mathbf{B_{\Gamma 1}} & -\dfrac{\mathbf{B_{\Gamma 1}K_u}}{\mathbf{1-K_u}} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix} + \begin{bmatrix} \mathbf{E_{11}} & \mathbf{E_{12}} & \mathbf{0} & \mathbf{0} \\ \mathbf{E_{21}} & \mathbf{E_{22}} & \mathbf{0} & \mathbf{0} \\ \mathbf{E_{31}} & \mathbf{E_{32}} & \mathbf{0} & \mathbf{0} \\ \mathbf{E_{41}} & \mathbf{E_{42}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B_{\Gamma 2}W_1} & \mathbf{B_{\Gamma 2}W_2} \end{bmatrix} \begin{bmatrix} \delta T_0(t) \\ \delta\emptyset_{free}(t) \\ \delta c_I(t) \\ \delta c_{Tar}(t) \end{bmatrix}. \quad (33)$$

$$\begin{bmatrix} y_T(t) \\ y_{soc}(t) \\ \delta\dot{\Gamma}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{C_{11}} & \mathbf{C_{12}} & \mathbf{C_{13}} & \mathbf{C_{14}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} T_{zone}(t) \\ \overline{T_{sl}}(t) \\ \overline{T_{se}}(t) \\ T_{im}(t) \\ x_{soc}(t) \\ \Gamma(t) \end{bmatrix} + \begin{bmatrix} \mathbf{D_{11}COP_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \dfrac{\mathbf{B_{\Gamma 1}}}{\mathbf{1-K_u}} & \mathbf{B_{\Gamma 1}} & -\dfrac{\mathbf{B_{\Gamma 1}K_u}}{\mathbf{1-K_u}} \end{bmatrix} \begin{bmatrix} u_{comp}(t) \\ v_1(t) \\ u_B(t) \end{bmatrix} + \begin{bmatrix} \mathbf{F_{11}} & \mathbf{F_{12}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B_{\Gamma 2}W_1} & \mathbf{B_{\Gamma 2}W_2} \end{bmatrix} \begin{bmatrix} \delta T_0(t) \\ \delta\emptyset_{free}(t) \\ \delta c_I(t) \\ \delta c_{Tar}(t) \end{bmatrix}. \quad (34)$$

where $\mathrm{A_{i-j}}$, $\mathrm{B_{i-1}}$, $\mathrm{C_{1-j}}$, $1 < \mathrm{i,j} < 4$ $\mathrm{D_{11}}$, $\mathrm{E_{11-42}}$ and $\mathrm{F_{11-12}}$ are the system matrices of the original IDEAS methodology.

Now that the state space equations have been formulated, the system can be put into MATLAB and Simulink where the OPTIC system can be simulated over a whole year. The values used within the simulation are as follows; $W_1 = W_2 = 0.5$, $\mathrm{c_I(0)} = 0.2$, $\mathrm{c_{Tar}(0)} = 0.3$, $\mathrm{v_1(0)} = 0$, $\mathrm{v_2(0)} = 0$ and $\mathrm{K_u} = 0.85$. Within the disturbance vector, ${}^{\Gamma}d(t)$, the $\delta c_I(t)$ component was collated from data provided by [8] in $\mathrm{gCO_2/kWh}$. Whilst the $\delta c_{Tar}(t)$ component has been kept at a constant value of $£0.3/\mathrm{kWh}$.The set point for temperature has been kept at 21°C but adheres to a standard occupancy pattern of heating 7am-9am and then 4pm-11pm with optimum start. The battery charge set point was set at 1kWh and the cost function was set to 0. The results of the stage three simulation can be seen in Fig. (9-13). The temperature yearly graphs show that during the heating periods, thermal comfort is maintained with some overheating in summer. This is not an issue with the OPTIC theory however, as it reflects the real-world building dynamics during the summer heating period. The battery rises to its set point of 1kWh and is then able to maintain its set point of the whole year. The cost function generally maintains its set point of Zero over the course of the year, but interestingly there are sharp spikes induced at the start and end of each heating period which are quickly resolved through the inverse dynamics control. Finally, the cost function rises over the course of the year in an expected manner. The whole year building simulation at 1-minute intervals successfully completed its run within 12.5 seconds making for an extremely fast building heating and energy demand dynamics model.

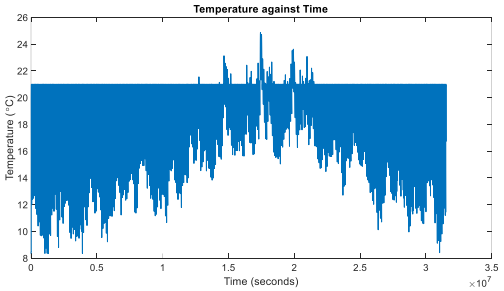

Figure 9- 1 Year Temperature (Degrees Celsius) against Time (Seconds) for OPTIC Stage 3.

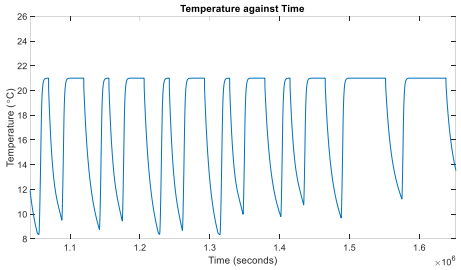

Figure 10- 1 Week Temperature (Degrees Celsius) against Time (Seconds) for OPTIC Stage 3.

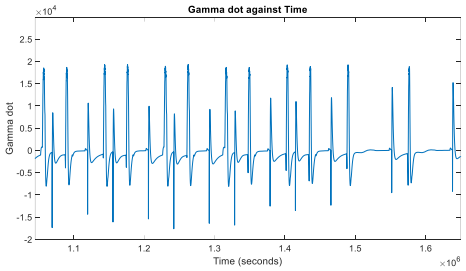

Figure 11- 1 Week $\dot{\Gamma}$ against Time (Seconds) for OPTIC Stage 3.

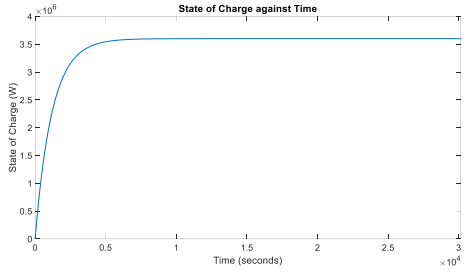

Figure 12- State of Charge (Joules) against Time (Seconds) for OPTIC Stage 3.

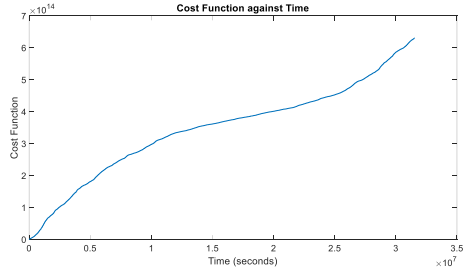

Figure 13- Cost Function against Time (Seconds) for OPTIC Stage 3.

IV. Results Discussion

In stage 1, the presented results laid the foundations for how the OPTIC methodology would work and integrate with the IDEAS methodology, inserting optimisation into the state space. The presented results showed that perfect temperature control was maintained for the simulation duration, whilst the additional OPTIC measured output of the $\dot{\Gamma}(t)$ vector performed as expected, attempting to reach its setpoint of 0 but not quite getting there due to temperature control needing to be maintained. As the complexity of the OPTIC method was increased to stage 2 to include the measuring of the state of charge, the temperature control of IDEAS was again, not disturbed, the response of $\dot{\Gamma}(t)$ was as expected and the battery control was able to reach its setpoint of 1kWh, and then maintain its setpoint without inducing any issues throughout the rest of the model. Finally, the OPTIC methodology underwent its final stage to include dynamic disturbances for the cost tariff and carbon intensity whilst also simulating over a full year at 1 minute intervals. The results of this stage showed to have been a success with the battery able to reach its set point and maintain stability, whilst the cost function $\dot{\Gamma}(t)$ output induced spikes at each switching point but then found stability as close to its Zero setpoint when possible.

V. Conclusions

The OPTIC cost function successfully allowed the thermal comfort to be accurately tracked whilst maintaining stability throughout all three stages. The OPTIC battery and cost function set points of Zero were also successfully maintained in their respective stages with none of the three set points showing any steady state error throughout the simulations. Next steps of this work will include a dynamic cost tariff and having dynamic battery set points. Furthermore, OPTIC is generic and can be applied to more complex heating systems, like a heat pump system. As a proof-of-methodology this initial research to develop OPTIC can be concluded to show promising results for future real heating system applications.

VI. References

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