

Technical Notation as a Tool for Basic Research in Relational Frame Theory

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Abstract

A core overarching aim of Relational Frame Theory (RFT) research on language and cognition is the prediction and influence of human behavior with precision, scope, and depth. However, the conceptualization and delineation of empirical investigations of higher-order language and cognition from a relational framing theoretical standpoint is a challenging task that requires a high degree of abstract reasoning and creativity. To that end, we propose using symbolic notation as seen in early RFT experimental literature as a possible functional-analytical tool to aid in the articulation of hypotheses and design of such experiments. In this article, we provide examples of aspects of cognition previously identified in RFT literature and how they can be articulated rather more concisely using technical notation than in-text illustration. We then provide a brief demonstration of the utility of notation by offering examples of several novel experiments and hypotheses in notation format. In two tables, we provide a “key” for understanding the technical notation written herein, which other basic-science researchers may decide to draw on in future. To conclude, this article is intended to be a useful resource to those who wish to carry out basic RFT research on complex language and cognition with greater technical clarity, precision, and broad scope.

Keywords: relational frame theory, basic research, notation, experimentation, precision, future research

Technical Notation as a Tool for Basic Research in Relational Frame Theory

We are at an interesting juncture in our capacity to provide parsimonious yet fruitful behavior-analytic accounts of verbal or symbolic behavior. In the present article, we highlight a somewhat neglected but highly pragmatic Relational Frame Theory (RFT; Hayes, Barnes-Holmes, & Roche, 2001) technical notation that may help interested researchers in the field to (1) formulate precise research questions and empirical designs, and (2) to avoid middle-level terms while conducting basic research. We then develop this notation to allow researchers to use it to (1) formulate testable hypotheses with precision, and (2) to design studies of complex cognition that are logical extensions of existing theory, but are difficult to clearly articulate in colloquial terms. Finally, we offer several concise novel hypotheses using this notation. This article is aimed at experimental researchers who program and run basic-science experiments rather than applied researchers concerned with testing “middle-level” applications of RFT. We anticipate that the use of this notation could increase the efficiency of scientific research communication and decrease the encroachment of middle-level terms within RFT laboratories when working out fundamental principles upon which therapies are based.

Language generativity and symbolism remain difficult challenges for behavior analysts to satisfactorily explain with traditional accounts (see Malott, 2003; Stewart, McElwee, & Ming, 2013). However, stimulus equivalence (SE; Sidman, 1971, 1994) and derived relational responding (e.g., RFT) have significantly contributed to our ability to understand, explain, predict, and influence higher-order cognition (see Dymond & Roche, 2013 for a book-length review). Within RFT, derived relational responding has occasionally been termed arbitrarily applicable relational responding (AARR). An apparent link between AARR and language (e.g., Devany, Hayes & Nelson, 1986; Dickins et al., 2001) provides evidence that symmetrical,

reflexive and transitive responding are features of language underlying its generativity and complexity. RFT has thereby allowed researchers to concisely define some of the most complex processes by which organisms adapt to their environments, thanks to the operational precision of behavior-analytic research.

The Need for More Basic Research

To date, both basic and applied AARR research has advanced our understanding of language and cognition in numerous ways. However, despite the acknowledged link between AARR and phenomena of practical interest (see Barnes-Holmes, Hussey, McEnteggart, Barnes-Holmes, & Foody, 2016; Cassidy, Roche, Colbert, Stewart, & Grey, 2016), it is increasingly apparent that our understanding of AARR is far from complete and it is necessary to elucidate the fundamental features and utility of AARR further (see Dymond, Roche, & Bennett, 2013). The need for further research is particularly acute for proponents of Acceptance and Commitment Therapy (ACT; Hayes, Strosahl, & Wilson, 1999) who regularly refer to the critical link between RFT and ACT (see Barnes-Holmes et al., 2016). It is proposed that RFT accounts for a number of core techniques or strategies employed by ACT therapists (e.g., heavy reliance on the use of analogies and metaphors; loosening rigid stimulus functions with cognitive defusion techniques). This link allows therapists to facilitate an expansion of a client's "psychological flexibility" or behavioral repertoire (see Blackledge & Drake, 2013). Indeed, there are putative claims that a therapy mainly based on RFT, known as Relational Frame Therapy (Törneke, 2010; Villatte, Villatte, & Hayes, 2015), might feasibly be developed. Accordingly, ACT (which, admittedly continues to grow in strength and popularity regardless) and Relational Frame Therapy would benefit from a similarly strong empirical base of basic and

applied research as that found in traditional behavior therapy (Blackledge & Drake, 2013; Dymond et al., 2013; Guinther & Dougher, 2015).

Middle-Level Terms

Middle-level terms (i.e., mid-level terms) are terms that appear to have face validity on the surface as technical operationally defined scientific terms in the clinical literature, but in fact are nontechnical terms (see Barnes-Holmes et al., 2016, for a thorough overview) because they do not help provide a precise functional account of clinical problems. In other words, middle-level terms might appear to be based in solid theoretical grounding, but they are regarded as nontechnical as they did not emerge, or were generated, from basic empirical research. Barnes-Holmes et al. helpfully provided examples of what might be considered a “low-level” term that emerged directly from basic scientific data (e.g., reinforcement) and a “high level” term that is somewhat abstract and atheoretical (e.g., attention). Middle-level terms in ACT include the six components of psychological flexibility (acceptance, cognitive defusion, self-as-context, present-moment awareness, values clarification, and committed action), and the broad overarching construct of psychological flexibility itself. The clinical literature typically treats the components of psychological flexibility and their interactions as functional behavioral processes (Barnes-Holmes et al., 2016). However, the use of such terminology in ACT and putative RFT accounts of problematic clinical behavior is an issue that is of current concern and debate (e.g., see Barnes-Holmes et al., 2018; Villatte et al., 2015). Indeed, in a review of Villatte et al.’s (2015) depiction of the relationship between ACT and RFT, Barnes-Holmes et al. (2018) highlighted their concerns with mixing technical and nontechnical terms in attempting to account for clinical phenomena. Mixing technical and nontechnical terms may give a false impression that all of the terms used have been operationally defined through experimentation. It should be

noted here, however, that Hayes, Barnes-Holmes, and Wilson (2012) have openly acknowledged the difficulty for ACT's middle-level terms to ever truly become, or be considered, basic functional technical terms, “. . . none of these are technical terms; none have the same degree of precision, scope, and depth of classical behavioral principles such as ‘reinforcement’, nor of theoretical RFT concepts such as the ‘transformation of stimulus functions. . .’” (p. 7; see also Barnes-Holmes et al., 2016).

Technical Notation

One tool presented in early RFT research (e.g., Steele & Hayes, 1991) that might be useful to revisit and revise, due to its potential benefit in helping researchers to become more precise and technical, is technical notation. Technical notation is used to achieve logical precision and clearer communication in subjects such as mathematics (see Peltomäki & Salakoski, 2004), computer science (Paternò, Mancini, & Meniconi, 1997), and even nanotechnology research (Leisner, Bleris, Lohmeuller, Xie, & Benenson, 2010). Notation appeared in some early texts presenting the core tenets of RFT, notably Hayes et al. (2001) and Steele and Hayes (1991), demonstrating that early founders of RFT supported the utility of technical notation. RFT notation is used to highlight contexts which might be used to predict *and influence* behavior with precision, scope, and depth. Previous appearances of this technical notation (e.g., Hayes et al., 2001; Steele & Hayes, 1991) may have appeared to be more arcane than functional. In recent years, it has not appeared in many publications as a result. However, now that researchers are investigating increasingly complex domains within basic RFT (e.g., Perez, Fidalgo, Kovac, & Nico, 2015), such as analogy (e.g., McLoughlin & Stewart, 2017), technical notation may help researchers to hypothesize about, explain, and explore such complex

AARR with increased precision. A compilation of key notation that may be of use to both experimenters and theorists within the field of RFT can be found in Table 1.

[Table 1 here]

Given that research has indicated that cognition is relational in nature (e.g., Cassidy, Roche, & Hayes, 2011; O’Hora et al., 2008), this notation syntax allows for a concise articulation of key concepts and hypotheses about AARR. Below, we describe some of the core features of language and cognition using notation.

Notation in Context

Mutual Entailment

$$C_{rel} (A r_x B) ||| (B r_y A)$$

The above notation indicates that within a given context (C_{rel}), if an organism has learned to treat the event A as having a relation (r_x) with B, then ($|||$) the organism should be able to derive that B is related to A in some way (r_y). One particular instance of mutual entailment is when the A:B relation is one of sameness. The relation of sameness is symmetrical, and thus it is possible to specify that $C_{rel} (A r_s B) ||| (B r_s A)$, where “ r_s ” is a relation of sameness. See Table 1 for a full summary of the notation used within this text. Table 2 illustrates possible variations of “ r_x ” notation in relation to some of the more commonly cited patterns of AARR.

[Table 2 here]

Combinatorial Entailment

$$C_{rel} (A r_x B); (B r_x C) ||| (A r_x C); (C r_y A)$$

This notation illustrates that in a specific context (C_{rel}), if an organism has learned to treat the event A as having a relation (r_x) with B, and the event B as having a relation (r_x) with C, then

(|||) the organism should be able to derive that A is related to C in the same way as A:B (r_x), and the mutually entailed relation ($C r_y A$).

This general pattern of AARR only applies to transitive relations (e.g., Slattery, Stewart, & O’Hora, 2011), for example larger/smaller, before/after, same/opposite, and so on (Johnson-Laird, 2010). There are indeed stimulus relationships dictated by the verbal community that do not necessarily lead to combinatorial entailment and these are labelled “intransitive.” For example, if A has met B and B has met C, it would not necessarily follow that A has met C. Using notation, this could be stated as:

$$C_{rel} (A r_{xi} B); (B r_{xi} C) |||$$

Here, “i” indicates that the relationship is intransitive.

Despite the illogicality of deriving ($A r_x C$) when considering an intransitive relationship, humans may still derive it. This overgeneralization error could in fact underpin some cognitive biases. For example, if Person A harms Person B, and Person B harms Person C, then Person C is not necessarily the victim of Person A. In some way, the existence of this intransitivity phenomenon presents a considerable theoretical and empirical challenge for RFT that is not readily accountable for in current formulations of the theory.

Likewise, if Class A and Class B are equivalence classes of people related via an asymmetrical relation (i.e., hierarchically) then the effect of Class A on Class B (e.g., oppression) does not necessarily hold for individual members (Persons A and B), as transitive class containment might suggest (see Slattery & Stewart, 2014). Such patterns of deriving false information may have an association with certain psychological disorders, for example, psychosis, anxiety, paranoia, and schizophrenia (see Stewart, Stewart, & Hughes, 2016).

Further, there are situations in which $(A r_x B)$ and $(B r_x C)$ can *never* derive $(A r_x C)$. For example, if A is the mother of B, and B is the mother of C, it follows that A can *never* be the mother of C. This is known as an antitransitive relationship, which can be notated as:

$$C_{rel} (A r_{xa} B); (B r_{xa} C) \parallel \parallel / (A r_x C)$$

Where “a” denotes anti-transitivity, and “/” denotes the lack of a derived relationship. In this instance, it would be functional *not* to derive $(A r_x C)$. As mentioned previously, RFT does not readily account for this type of relation. As (we hope) RFT grows to account for these unexamined types of relationships, it would be useful to use notation, because various kinds of relationships can be written in such notation easily.

Accordingly, basic transitive combinatorial relations would be written in notation as:

$$C_{rel} (A r_{xt} B); (B r_{xt} C) \parallel \parallel (A r_{xt} C); (C r_{yt} A)$$

Here, “t” denotes transitivity.

Networks containing transitive relationships quickly expand with the addition of further stimulus relationships. For example, consider that training three relationships $(A r_{xt} B)$, $(B r_{xt} C)$, $(C r_{xt} D)$ combinatorially entails as such:

$$C_{rel} (A r_{xt} B); (B r_{xt} C); (C r_{xt} D) \parallel \parallel (A r_{xt} C); (C r_{yt} A); (B r_{xt} D); (D r_{yt} B); (A r_{xt} D); (D r_{yt} A)$$

If we train a five-node network with four stimulus relations A-B-C-D-E, this combinatorially entails six relationships (if we decline to count the mutually entailed relations of those directly trained). If we take into account the mutual entailments of each of the combinatorially derived relations, 12 stimulus relations are derived:

$$C_{rel} (A r_{xt} B); (B r_{xt} C); (C r_{xt} D); (D r_{xt} E) \parallel \parallel (A r_{xt} C); (C r_{yt} A); (A r_{xt} D); (D r_{yt} A); (A r_{xt} E); (E r_{yt} A); (B r_{xt} D); (D r_{yt} B); (B r_{xt} E); (E r_{yt} B); (C r_{xt} E); (E r_{yt} C);$$

This notation demonstrates the generativity inherent in relational behavior.

Transformation of Functions

$$C_{\text{func}} [C_{\text{rel}} \{(A r_x B; B r_x C) \parallel (A r_x C; C r_y A)\}; \{Af^1; Bf^n; Cf^m\} \parallel (Bf^2; Cf^3)]$$

Transformation of stimulus functions occurs when contextual contingencies select a behavioral function or value. This statement says: In a given context (C_{rel}), if the organism has derived a relation ($A r_{xt} B; B r_{xt} C \parallel (A r_x C; C r_y A)$) and a nonrelational function of a stimulus in that relational network (e.g., Af^1) has been established in the organism's behavioral repertoire (C_{func}), then (\parallel) the organism will derive the relative functions of stimuli participating in the relational response (i.e., the functions of B and C are modified based on the relations in which they participate).

Analogical Relations

$$C_{\text{rel}} (A r_x B); (C r_x D) \parallel (A:B) r_s (C:D)$$

This notation specifies that within a particular context (C_{rel}), if two relations (i.e., A:B and C:D) are of the same type (i.e., r_x and r_x), then a relation of coordination or functional equivalence (r_s) might be derived between these relations.

Differentiated Relations

$$C_{\text{rel}} (A r_x B); (C r_y D) \parallel (A:B) r_d (C:D)$$

The above notation expresses that within a given context (C_{rel}), if two relations (i.e., A:B and C:D) are of differing types (e.g., r_x and r_y), then a relation of distinction (r_d) might be derived between these relations.

It is possible to use notation to identify increasingly complex kinds of relational responding that might be tested and/or trained. For example:

$$C_{\text{rel}} (A r_s B); (C r_o D) \parallel (A:B) r_d; r_o (C:D)$$

The above notation illustrates that within a specific context, if an organism treats two stimuli (A and B) as being the same (r_s), and two more stimuli (C and D) as being opposite (r_o), then it might be derived that (\parallel) the relation between the first relation (A:B) and the second (C:D) is one of difference (r_d), specifically opposition (r_o). It should be noted that if notation was not used for the examples above, it would likely have taken hundreds more words and dozens of potentially ambiguous or easily misinterpretable diagrams to explain the stimulus relations in question.

Some More Future Studies

It is possible to include a plethora of relations in studies of complex cognition. For example, below we include hypotheses pertaining to differentiating r_d from r_o , and r_b (before) from r_a (after):

$$C_{rel} (A r_d B); (C r_o D) \parallel (A:B) r_d / r_o (C:D)$$

$$C_{rel} (A r_b B); (C r_a D) \parallel (A:B) r_d; r_o (C:D)$$

The utility of establishing such fine experimental control over AARR is an empirical matter (e.g., these kinds of skills trained to fluency may be useful for mathematics and other forms of higher logic). In this instance, the term “behavioral fluency” refers to the combination of precise and swift responding that is considered to be synonymous with expert performance or mastery of a behavioral repertoire (Binder, Haughton, & Bateman, 2002; McTiernan, Holloway, Healy, & Hogan, 2015; Ramey et al., 2016). The concision offered by such notation may allow for clearer prediction and influence over increasingly complex behaviors in future, including behaviors that are perhaps currently beyond our species.

It may be possible to investigate the derivation of relations within yet more complex relations. For example, below are two competing hypotheses asking whether individuals will

derive a “more than” relation to be opposite a “less than” relation, or simply consider them distinct.

$$C_{rel} (A r_m B); (C r_l D) ||| (A:B) r_d (C:D)$$

$$C_{rel} (A r_m B); (C r_l D) ||| (A:B) r_o (C:D)$$

There are many other nonsymmetrical relations that could be similarly related. For example, the hypotheses below ask: is “before” opposite to “after,” or just different?

$$C_{rel} (A r_b B); (C r_a D) ||| (A:B) r_d (C:D)$$

$$C_{rel} (A r_b B); (C r_a D) ||| (A:B) r_o (C:D)$$

These may have useful applications for the understanding of complex phenomena. For example, RFT considers “the self” to be a nexus of many established relational networks. To differentiate relational networks may therefore be an important skill underlying the ability to differentiate among different “selves.” Learning to do this expressively (e.g., McLoughlin & Stewart, 2017, modeled this behavior of differentiating relational networks receptively using the Relational Evaluation Procedure) could constitute an operationalized account of “I–you” relational framing. With the inherent complexity that comes with such novel questions, notation could ensure the technicality and precision of hypotheses and experimental procedures, while simultaneously allowing them to be communicated concisely.

Likewise, complex relational repertoires, such as that of hierarchical classification are amenable to RFT notation. For example, below we include a rudimentary notation of hierarchical classification:

$$C_{rel} (A1 r_p B; A2 r_p B; B r_p C) ||| (A1 r_s A2; A1 r_p C; A2 r_p C; C r_c A1; C r_c A2; C r_c B; B r_c A1; B r_c A2)$$

This notation outlines that the stimuli A1 and A2 are part of the stimulus class B, which is part of the stimulus class C. From this information, an individual can derive a degree of functional sameness between stimuli A1 and A2, while simultaneously deriving that the stimuli A1 and A2 are part of the stimulus class C. This also leads to further derivations including that stimulus class C contains stimuli A1, A2, and B, while the stimulus class B contains the stimuli A1 and A2. The complexity of these relational networks are further outlined when the transformation of stimulus function is considered.

$$C_{\text{func}} [C_{\text{rel}} \{(A \text{ r}_c B; B \text{ r}_c C) \parallel (A \text{ r}_c C; C \text{ r}_p A; C \text{ r}_p B; B \text{ r}_p A)\};$$

If we derive “A contains B, and B contains C” for functional purposes, then

$$\{A^f; B^m; C^m\} \parallel (B^f; C^f);$$

That is, if we know the functions of superordinate class A, they will change the functions of subordinate classes B and C. Furthermore,

$$\{A^m; B^f; C^m\} \parallel (A^m; C^f);$$

If we only knew the function of class B, it would transform the functions of subordinate class C and not superordinate class A. Finally,

$$\{A^m; B^m; C^f\} \parallel (A^m; B^m)$$

If we only knew the function of subordinate class/stimulus C, then it does not necessarily tell us about the functions of its superordinate classes A and B. That is, we might expect all of the functions of the containing network to transform the functions of the member network, but the member network should not transform the functions of the class. This is, of course, a testable hypothesis. It is also possible that the salient functions of a group are abstractions of what's common across its constituents, which might mean that individual members transform functions of the group as a whole. This can be good, because it is useful to know the truth criteria for

category membership; categories help us to simplify the world around us. In other cases, perhaps, this may not be so adaptive. For example, I might think that a key feature of what it means to be a RAEF (superordinate group) is that they have DOBs. Then I might hear about individual RAEFs (named Jeff, Toby, and Ben): Ben explains things condescendingly; Toby is unfaithful to his partner; Ben is prejudiced against non-RAEFs. Being mean and undesirable is common across Jeff, Toby, and Ben, and so it is possible that individuals transform the functions of groups for the worse. In other words, I might now generalize from these exceptional exemplars to say that RAEFs are mean-spirited (or worse), and conclude that we need to create quotas of non-RAEFs to keep them in check. The problem here would be that any person who has a DOB and, therefore, fits into the category “RAEF,” or demonstrates any otherwise advantageous trait associated with being RAEF-like, may be stereotyped as being like Jeff, Toby, and Ben. This would be a logical non sequitur, and potentially obscure the fact that, in some contexts, it’s good to be RAEF-like. If the hypothesis in the notation above were to be rejected in an experiment (i.e., if participants derived functions of a category that don’t generally apply to it), the errors may be indicative of a deficit in hierarchical AARR abilities, and so training these generalizable patterns of behavior could be justified. Indeed, there are precedents in the literature for training both simple AARR (e.g., Cassidy et al., 2011) and more complex AARR repertoires (e.g., McLoughlin, Tyndall, & Pereira, 2018; see also Guinther, 2018).

We may also refine more basic assumptions using empirical tests. For example, does transformation of stimulus functions always happen as expected across a combinatorial relation? It is possible to conceive of an instance when it does not.

$$C_{rel} (A r_x B); (B r_x C) ||| (A r_x C); (C r_y A)$$

In the above account of combinatorial entailment, the relation between A and B (r_x) is the same as the B:C relation. This entails that the A:C relation should be the same as the A:B and B:C relations, apart from by order of magnitude. However, as has been shown, a relationship may be transitive or intransitive, and combinatorial entailment should only occur in the former. Perhaps this explains why some participants do not always combinatorially entail in studies of this nature (e.g., Quinones & Hayes, 2014). A future experiment could examine whether participants could be influenced to treat relationships as transitive or intransitive. For example, participants could be repeatedly trained on relationships such as:

$$C_{rel} (A r_{xa} B); (B r_{xa} C) ||| / (A r_x C)$$

or

$$C_{rel} (A r_{xi} B); (B r_{xi} C) |||$$

If participants could be so influenced, it may be possible to train participants not to combinatorially entail so readily, which would lead to patterns of relational framing that are more selectively applied, and in doing so this could prevent the spread of negative stimulus functions through overgeneralization and reduce cognitive errors.

Concluding Remarks

Of the few *principles* in psychology, behavioral selection by consequences is arguably the most fundamental offered by the field. There appears to be somewhat of a converging consensus from various fields, including behavioral psychology (Hayes et al., 2001), cognitive psychology (Halford, Wilson, & Phillips, 2010), and linguistics (Garcia, 2015), that indicates that language and cognition are relational in nature, with increasingly complex language and cognition involving the utilization of progressively complex relational responses (see Barnes-Holmes et al., 2005; Cassidy et al., 2011; Cassidy, Roche, Colbert, Stewart, & Grey, 2016;

Hayes & Stewart, 2016; Moran, Walsh, Stewart, McGhee, & Ming, 2015; O’Hora, Barnes-Holmes, Roche, & Smeets, 2004; Stewart, Barnes-Holmes, & Roche, 2004). It should be acknowledged that RFT as a functional-analytic account of human language and cognition (Hayes & Barnes, 1997) has shed a considerable amount of light on complex relational processing in a relatively short period of time. As this field develops, it may become difficult to articulate hypotheses in-text as the kinds of high-level operant responses trained and tested incrementally become more complex.

In this short article, we have proposed that the notation style appearing in early RFT literature may be useful in that regard. In addition, we have attempted to illustrate its utility using multiple exemplars and have provided a “key” (see Tables 1 and 2) regarding some useful notation for formulating hypotheses for exploring AARR. It is salient that the use of such notation might also conceivably assist in the design of experiments that may help to counteract research that claims empirical findings in RFT studies can generally be accounted for by appealing to contextual control of equivalence relations alone (e.g., Sidman, 1994; Alonso-Álvarez & Pérez-González, 2017). For example, Alonso-Álvarez and Pérez-González (2017) proposed that prior RFT empirical demonstrations of derivations of “Same” and “Opposite” relations (e.g., Dymond & Barnes, 1995; Dymond, Roche, Forsyth, Whelan, & Rhoden, 2007; Whelan & Barnes-Holmes, 2004) could also be explained by contextual control over equivalence and nonequivalence relations, respectively. Although Alonso-Álvarez and Pérez-González’s proposal might have immediate appeal in terms of simplicity and parsimony, it is difficult to conceive how their position might potentially account for many of the more complex AARR relations that the use of RFT’s technical notation might predict.

For translational RFT-to-practice researchers targeting complex repertoires such as perspective-taking or psychological flexibility, perhaps the challenge of operationalizing such concepts in more technical terms might help them to identify relevant manipulable behavioral contingencies. However, as noted above, at present middle-level terms such as “psychological flexibility” have not yet been clearly operationalized in technical terms, but in colloquial terms (Barnes-Holmes et al., 2016). This is not to say that mid-level terms such as these are not useful in certain contexts, but they may warrant further exploration given that these are concepts upon which many practitioners (e.g., ACT therapists) base their practice, rather than principles that survive through basic experimentation.

In summary, scientists studying AARR may be able to use this notation to communicate increasingly complex hypotheses with precision, as complexities in experiments evolve. Naturally, this notation remains one of the more arcane aspects of RFT and may not have initial appeal to a casual or applied practice readership. It is important to acknowledge that from an RFT perspective all definitions will be judged ultimately by their utility. Thus, our goal is not to test RFT predictions per se or even provide tools for assessing the coherence of the definitions with logical notation but rather to put notation on RFT as it was originally proposed. Nonetheless, technical notation can prove extremely useful to basic RFT researchers for the formulation of succinct hypotheses, particularly in relation to complex cognition. Technical notation may also provide clarity in terms of communicating AARR research to those who are inclined to engage with RFT at the basic science level. This piece is intended to function as a nondefinitive, but useful resource in that regard.

Compliance with Ethical Standards

Disclosure of Conflicts of Interest: On behalf of all the authors the corresponding author confirms that no author has a conflict of interest to declare.

Funding: The study was not supported by any grant funding from any institution or organization.

Ethical Approval: Not applicable as not an empirical study paper.

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Table 1

Relational Frame Theory Notation

Syntax	Meaning
C_{rel}	Contextual contingencies selecting a relational response
C_{func}	Contextual contingencies selecting a behavioral function
r	A relation
f	A stimulus function
f^n	The unspecified stimulus function “n” (superscript, specified via numeric characters*)
A:B	An undefined relation between two stimuli, “A” and “B”; “A is to B”
r_x	The undefined relation “x” (specified via alpha characters)
r_y	An undefined relation that is not necessarily “x,” only used after “ r_x ”
	“Entails,” or “predicts”
X	The stimulus “X”
Xf^n	The unspecified function of stimulus “X” (superscript, specified via numeric characters)
;	“And”
/	“Not,” or “but not” (e.g., “A r_d / r_o B” means “A is different from but not opposite to B”)

Table 2

Notation for Common Patterns of Arbitrarily Applicable Relational Responding

Syntax	Meaning
r_s	A coordinate (functional sameness) relation
r_d	A distinction (functional difference) relation
r_o	An opposition relation
r_b	A “before” temporal relation
r_a	An “after” temporal relation
r_m	A “more than” (or “greater than”) comparison relation
r_l	A “less than” comparison relation
r_p	A “part of” hierarchical relation
r_c	A “contains” hierarchical relation
r_x^{+n}	Used to emphasize comparative relationality (e.g., “ r_m^{+1} ” could mean an “even more than” relation)
r_{xi}	Specifies that this particular relational cue “x” is intransitive
r_{xa}	Specifies that this particular relational cue “x” is antitransitive
r_{xt}	Specifies that this particular relational cue “x” is transitive