Learning to Teach Mathematics

Navigating the Landscape of Teacher Education

Thesis submitted in accordance with the requirements of the University of Chester for the degree of Doctor of Education

by Sally Bamber

July 2015

Learning to teach mathematics: navigating the landscape of teacher education

I declare that the material being presented for examination in this thesis is my own work and has not been submitted for an award of this or another Higher Education Institution.

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July 2015

Doctor of Education

Acknowledgement

I would like to thank Jeff Adams and Anne-Marie Wright because I have been supported, guided and inspired by two superb supervisors throughout my Doctoral Thesis. I am grateful to all of the administrators and teachers in the Education Doctorate programme at the University of Chester for their guidance and encouragement. I am fortunate to have been sponsored by the Faculty of Education at the University of Chester throughout this degree.

I am indebted to five wonderful teacher educators; Martin Bamber, Shaine Bushell, Ann Cook, Barbara Craig and Bethan Hulse for their patience, inspiration, humour and proof-reading and for having faith in my research. Finally, I am grateful to the past and present PGCE Mathematics teachers who have taught me about teacher education and who have shared their experiences with me to allow me to tell their stories.

Dedication

For Three Things: Joe, Tom and Evie

You now have my undivided attention

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Sally Bamber

Abstract

Metaphor provides a potentially powerful rhetorical device to help me to tell informed and persuasive stories about mathematics education. In this ethnographic study I consider key episodes that serve to exemplify the complex experience of Initial Teacher Education (ITE) students of secondary mathematics education. I use a narrative analysis to shine a spotlight on the experiences of six beginning teachers so that the metaphors in their stories expose the impact that separately situated sites of teacher education have upon their beliefs and behaviour as teachers. Tensions between school and university contributors to teacher education have been well documented over many decades, but recent policy changes in the nature of post-graduate ITE in England bring these issues to the fore. In this study, I consider the influences of school-based and university-based teacher educators upon the beliefs of student secondary mathematics teachers and interpret the students' perceptions of these influences on their actions as novice teachers. My analysis is framed by a model of experience and education articulated by Dewey as well as a framework of representations of knowledge in a culture of education articulated by theorists concerned with the relevance of constructivism and situated cognition as theories of learning. In this study, disturbances and discontinuities relating to the location and culture of ITE, together with the development of ITE students' professional knowledge are uncovered, warranting further research.

Summary of Portfolio

Research Methodologies for Professional Enquiry

This interpretive study explored appropriate methodologies to research mature student teachers' perceptions of their beliefs about mathematics teaching and learning. In doing so, a narrative analysis was used to study one student mathematics teacher's perceptions of his emerging teacher identity and his perception of how his beliefs about mathematics education changed during his Initial Teacher Education (ITE) course. This study exposed a number of contemporary issues in mathematics education and found that predominantly qualitative data were appropriate for this interpretive research, rejecting grounded theory and quantitative methods.

Social Theory and Education

This discussion applied the theories and philosophy of John Dewey and Paulo Freire to an ethnographic study of two ITE mathematics students' beliefs about mathematics teaching and their aspirations as novice teachers. The study explored how the students' beliefs and aspirations were influenced by their school-based teaching experiences. Through scrutiny of Dewey's theory in relation to Democracy and Education, Dewey's theory of Experience and Education emerged as the most appropriate framework for the analysis of students' perceptions of their teacher education as well as their interpretation of the mathematics education of their pupils. Parallels were drawn between the theories of Freire and Dewey in the context of secondary mathematics education.

Creativity in Practice

This mixed methods study explored parallels between mathematics and mathematics education through an exploration of connections evolved from two teaching interventions. Connections in number theory, serving to exemplify the elegance of patterns within agreed mathematical constructs, were juxtaposed with qualitative data from mathematics teaching interventions. Through this, unanticipated connections between potentially opposing aspects of professional knowledge evolved in teacher educators' interpretations of the data.

Policy Analysis

This policy analysis was constructed from my interpretation of the impact of the Department for Education's White Paper, *The Importance of Teaching* and the subsequent reconstruction of *Teachers' Standards* in 2012 on ITE practices. The implications of these policy changes upon ITE students was discussed in the analysis.

Institutions, Discontinuities and Systems of Thought

Cartesian and Hobbesian interpretations of the nature of knowledge were used to analyse the institution of secondary school algebra and the consequences of these interpretations for one ITE mathematics student. Empirical data was gathered that exposed discontinuities in perceptions of secondary school algebra that were interpreted using Kristeva's notion of abjection and through contrasts derived from differences in rationalist and empiricist systems of thought.

Chapter 1 Introduction

The landscape of post-graduate Initial Teacher Education (ITE) in England is currently in a period of change as traditional university led ITE declines and school led provision grows with the emergence of School Direct ITE (DfE, 2010, 2011). Beginning teachers of secondary mathematics are situated, or perhaps caught within political and policy flux (Furlong, 2005, 2013). This study shines a spotlight on six beginning teachers, telling the stories of their experiences of mathematics teacher education at the start of a period of potential transition in ITE.

I have chosen to tell these students' stories at this time because I am sensitised to the impact of cultural, social and political influences on my students' professional knowledge and their resulting behaviour in secondary mathematics classrooms. My students are influenced by neoliberal reforms, in a climate of performativity that is dominated by the impact of local interpretations of what inspectors, led by the Office for Standards in Education (Ofsted), are looking for (Ball, 2003). This influence permeates activity in school and university and, as my study will demonstrate, has a direct effect on my students' perceptions of learning to teach mathematics. I believe that my students have a story that should be told so that teacher educators and mathematics teachers understand my students' experience of learning to teach mathematics at the start of this period of policy reform. My experience as a secondary mathematics teacher, mathematics curriculum leader, teacher educator and beginning university researcher equip me to tell it.

PGCE secondary mathematics students are at the heart of this study as I establish the impact of my behaviour and the behaviour of the teachers in school upon their beliefs about mathematics education. I focus the attention of my research on their perceptions of their actions in secondary mathematics classrooms because I want to understand the PGCE students' interpretation of guidance from me and from school-based mentors about how they should plan and teach mathematics to 11-16 year olds. The aims of this study are addressed in the following research questions:

- How do student mathematics teachers articulate their perceptions of their experiences of learning to teach mathematics at a time of policy changes in ITE?
- What are student mathematics teachers' perceptions of university-led and school-led ITE?
- What are student mathematics teachers' beliefs about mathematics education?
- How do student mathematics teachers justify their actions in secondary mathematics classrooms?

To address these questions, I do not present a simplistic dialectic between my university teaching and school experience because my analysis of my students' professional learning is far more complicated than this dialectic suggests. Hence, my interpretation of my students' professional learning is underpinned by Wenger's concept of landscapes of learning (1998, 2013) in order to understand the many influences on PGCE students during their initial teacher education.

The themes analysed are characterised by disturbances and discontinuities that influence my students and originate in cultural, political, social and contextual issues that occur on a local, national and even global scale. Furlong (2005, 2013) cites the impact of government policies (DfE, 1998) upon teachers' professionalism by redefining the professional teacher such that "*modern teachers needed to accept accountability; take personal and collective responsibility for improving their skills and subject knowledge; seek to base decisions on evidence of what works in schools in this country and internationally*" (Furlong, 2013, p. 34). In his interpretation of the teaching professional, policies suggest that a teacher operates in the pursuit of implementing practices deemed by superiors as valuable because they are perceived to have 'worked' to raise attainment in another setting. However, the neoliberal notion of collective responsibility within these policies is absent because individualisation is dominant within the subject's individual responsibility for his or her own 'performance' in school (Ball, 2003, 2013). Through this, teachers are obliged to relinquish personal responsibility for their professional knowledge and expertise, relinquish their own sense of agency in developing their professional autonomy, in favour of an externally managed and nationally defined structure of professional capability. Changes to the nature of ITE in England, have resulted in the current climate of teacher education that diminishes the notion that universities and schools may both have a contribution to make to teacher education because, "*the idea that universities and schools might have different perspectives and different forms of knowledge to contribute to teacher education was increasingly squeezed out"* (Furlong, 2013, p. 37). This suggests that research informed practice located in universities is not valued, to be replaced by 'what works' practice located in school and culturally reproduced between teacher and teacher from school to school.

Jones (2013) argues that the influence of education policy on practice is more complicated than to suggest that the current performance-driven culture in schools is a direct consequence of the neoliberal interpretation that is popular in much western education policy debate. She argues that education policy is controlled by government, but that these policies become drenched in the values that characterise those with a direct influence on practice. Thus, local interpretation of policy by school communities, teachers, school leaders and parents is influenced by the values that characterise these communities so that the way that policy develops in action in schools is complex and cannot merely be viewed as a direct consequence of education policy from government. Within this study, my students' perceptions of local interpretations of the external influence of education policy will be analysed and discussed from the perspective that my students' realisation of the influence of policies on their practice is subject to local understanding of that policy in their schools. At the time of this study, education reforms introduced by the coalition government's first secretary of state for education were described by Ball (2013) as giving teachers freedom, but that freedom was premised by the freedom to work on 'what matters', where 'what matters' is determined by the secretary of state for education. Although my study is conducted within the confines of the PGCE course that I teach, the influence of government policy on the schools where my students learn to teach serves to influence and complicate their beliefs and actions.

Ball and Olmedo (2013) account for the individualisation of teaching within neoliberal education that re-articulates teaching as a set of shallow skills and competences aligned to measures of performance and effectiveness that are subsequently aligned to the accountability measures that define teacher competencies. Through this re-articulation the classroom teacher is responsive to external influences rather that reflective at the site of teaching and learning. Ball and Olmedo argue that teachers are oppressed by neoliberal education, but are also the product of it, observed through their perceptions, relationships and behaviour in school. This argument is apparent within this study, seen through my students' articulation of influences upon what they believe and how they act while learning to teach mathematics.

This phenomenon became most apparent to me in 2005 when I returned from teaching in Hong Kong to work in an English secondary school. When I left England in 1999, my school was coming to terms with the expectations of the national strategies and accountability measures that were being disseminated throughout England. At that time, I did not perceived the full impact of external measures of accountability and the individualisation of the role if the teacher (Ball, 2003) upon my school and my colleagues. Justification for developments in the classroom was largely focussed on the experience of the pupils that we taught and their attainment in a range of subjects. Early attempts at target setting, for both teachers and pupils, had crept into our roles, but without the fear that the consequences of our actions in relation to targets would define our perceived success as teachers and our pupils as learners. Upon returning in 2005, I was immediately struck by the change in culture in the school where I taught. Without exception, justification for all initiatives and developments were located in the rhetoric of 'this is what Ofsted are looking for'. In this respect, teachers and their roles were defined by the school leaders' perceptions of the Ofsted framework for inspections as much as by the proportion of pupils who gained five or more GCSE grades C to A*. It appeared to me that teachers were becoming the product of local interpretation of external regulation (Gill, 2008) that changed the relationship of my new colleagues with other teachers, with their pupils and also with themselves. Soon after, I became a teacher educator based in a university, where I have observed mathematics departments subject their GCSE pupils to repeated cycles of GCSE examination and to extensive intervention classes during pastoral periods so that more and more pupils may achieve expected levels of progress through whatever examination related means necessary. My students are influenced by these activities in the schools where they learn to teach, which has the potential to directly influence their perceptions of what it means to teach mathematics and to have a direct consequence upon their behaviour in the mathematics classroom.

I interpret my influence upon my students' beliefs and the actions observed in their classrooms through a model of dominant teacher orientations present in school, which is derived from the work of Askew (Askew et al., 1997; Askew, 2002) and Swan (2005). I teach PGCE students a connected model of mathematics teaching, whilst they report that they are exposed to a predominantly transmission model during their school experience (Ofsted, 2012). The transmission teacher orientation is congruent with traditional models of education that position the teacher as the focus of knowledge that is delivered to students through ready-made definitions and exercises for students to practice. Alternatively, in the connectionist orientation, the teacher chooses the influences that he or she believes that the students need to stimulate connections between their existing knowledge or understanding and new learning. These models are relatively crude because transmission and connected models of teaching are not mutually exclusive, nor do they allude to the perceived political pressures that student and experienced teachers report in their professional roles (Ball, 2003, 2006; Lerman, 2014). Teachers may believe in one model of mathematics teaching, but operate in a complicated political, social and emotional landscape that influences their actions, leading to a model of teaching that is less aligned to their beliefs than they might have aspired to when entering the profession. I interpret aspects of this ITE landscape through my participant observations of their teaching, my teaching, field notes from lesson observation discussions and semi-structured interviews.

One further complication is the manner in which student teachers' professional knowledge and their perceptions of the knowledge of the people that influence their teaching is articulated by me and by my students. I use Shulman's (1987) model of teacher knowledge to interpret the student teachers' beliefs about their development in relation to their pedagogical mathematical knowledge and their knowledge of the teaching context. This knowledge is confined to their placement school, but influenced by their experience of other schools. I use the terms 'pedagogical mathematical knowledge' to correspond with Shulman's definition of 'pedagogical content knowledge', which I summarise to my students as the ability to make mathematics learnable. Predominantly, explicit teaching that addresses their pedagogical mathematical knowledge is the focus of the university-based teaching that they receive from me, while knowledge of the context for teaching comes from the communities of practitioners that they work with in school. My analysis of the influences on the PGCE students' perceptions of these two kinds of knowledge is rich with critical incidents that I explore further with my students both as a group and individually. The landscape of learning (Wenger, 2013) shifted more rapidly during the two years that I gathered data for School Direct students, although the influence of policy changes is apparent for all ITE students (Ball, 2003; 2013; Furlong, 2013). School Direct students' ITE providers are the lead schools within which they apply to. However, until schools are accredited as ITE providers by the Department for Education, aspects of the School Direct students' teacher education are provided by an accredited provider, such as the university where I teach. With respect to assessment and mathematics teacher education, School Direct students and traditional PGCE students have comparable experiences in either route. Therefore, participants in this study come from the traditional PGCE as well as the School Direct route. However, the nature of School Direct participants' experience of the context in which they teach is distinct from PGCE students following a traditional training route because the School Direct students are aligned to their training school more explicitly than traditional PGCE students (DfE, 2011) and, therefore, these two groups are analysed separately in this study.

My analysis of student teachers' mathematical pedagogical knowledge includes their perception and interpretation of different representations of mathematics using Bruner's model of Enactive-Iconic-Symbolic (EIS) representations (1966). Typically, secondary mathematics PGCE students enter ITE courses with a good understanding of symbolic representations of mathematics and are then exposed to enactive and iconic representations in the university mathematics education classroom in order to analyse the way that mathematics is learned by the pupils in their classrooms. The ITE students' perceptions of their experience of these representations is explored and analysed within this study.

In a similar manner, my students' perceptions of how they are learning to teach as well as how their pupils are learning mathematics is a feature of my research. In this respect, the theoretical framework for my analysis of both the mathematics classroom and the teacher education landscape draws upon a model of democratic education described in Dewey's theory of experience and education (1938), which is discussed in the following chapter.

The context for selecting participants and the framework for gathering data in this study has been fluid because my students' stories evolved in relation to critical incidents within their ITE year, which in turn evolved into my narrative analysis of my students' experience through a series of key episodes. Frequently, these episodes have been stimulated by metaphors and analogies used by my students or their mentors. The nature of the metaphors used in each episode provided me with a stimulus to explore themes in semi-structured interviews. The richness of the data that my students allowed me to gather exposes their specific experiences and individual perceptions in detail.

Initially, I did not design the study to capture metaphors to make meaning from my students narratives, but this is how the study has evolved because of the way they told me their stories. Lakoff and Johnson argue that "*metaphor provides a way of* partially *communicating unshared experiences, and it is the natural structure of our experience that makes this possible"* (2003, p. 225) to justify the use of metaphor, not just as a rhetorical device, but as a way of understanding concepts through the language that describes the concepts. Metaphor is used in this manner in my study, exemplified through one participants use of the phrase “*caught between divorcing parents*” to describe her perception of messages from school-based educators and university tutors. Through this metaphor, the participant and I understand her experience because we share understanding of the literal concept of a child caught between divorcing parents so that together, we can understand the antagonistic tensions that she has experienced during her teacher education, understanding that her allegiance is pulled in one direction by messages from school-based educators and then in another by university-based educators’ messages.

I am in the position of capturing these stories ethnographically because of my distinctive position as my students’ university tutor. I am able to bridge the ‘*parallel universes’* described by one participant, as I am able to share experiences with the students at school and university and because I have established relationships with the ‘players’ in their school experiences. It is this privileged position that leads me to justify my reflexive stance within my ethnographic study. Only by operating within the landscape in which my students learn, can I truly understand the nature of the phenomena that I am studying. I am on the periphery of many of the experiences that my students have, but my own experience as a mathematics teacher and mathematics teacher educator means that I am drenched in an understanding of the context of learning to teach in the classroom and within the university. Hence, I have adopted an ethnographic approach to my study.

I use Pillow’s (2003) justification of reflexivity as a powerful methodological tool, not to justify a self-indulgent exploration of what I do and who I do it to, but to allow for a better view of student teachers' perceptions and beliefs because of my position within the phenomena that I am studying. The potentially disturbing ethical position that my proximity might create has been mitigated by informed and often enthusiastic consent of all those involved in the study as well as my careful adherence to Clough and Nutbrown's notion of radical listening (2007) as I tell the stories of my students.

Through this analysis I have identified disturbances in student teachers' experiences of learning to teach mathematics that relate to my research questions. With respect to how my students articulate their experiences of learning to teach at a time of policy changes within ITE, it is clear that the location of professional learning that facilitates or stifles critical reflection on practice influences their experiences. Similarly, the situated nature of learning to teach in relation to the influence of the context and culture of the school on my students' professional learning has an impact on my students' perceptions of school-led and university-led ITE. In particular, my students' perceptions of the incompatibility of the pedagogy located in university and school is a theme that runs through this study. With respect to my research question, asking how my students justify their actions in the secondary school classrooms two themes emerge. Firstly, the point at which my students' fascination with how they teach transforms into sensitivity to how their pupils' learn effects their justifications of their actions in the classroom. Secondly, my students' description of their behaviour in relation to their professional knowledge exposes justifications for their actions that are inconsistent with their beliefs, but consistent with discontinuities between university and school led ITE.

In order to identify an appropriate theoretical framework for interpreting the themes identified above, theories of learning are discussed in Chapter 2. These are the theories that influence my teaching, my interpretation of my students' learning and my interpretation of the pupils' learning that I observe in PGCE students' classrooms.

Chapter 3, which follows my theoretical discussion, identifies the methodological considerations that have informed my research, integrated with a justification for my ethical approach to this study. This includes my rationale for using reflexivity and ethnography as methodological tools in my research.

The data that informs this study is presented in four chapters of episodes that illustrate six student teachers' beliefs about how they learn to teach mathematics and their perceptions of the influence of university-based and school-based teacher education on their actions in their classrooms, as well as the influence on their evolving professional knowledge. The episodes are structured as follows;

- Episode 1 tells the story of two PGCE students following the traditional university-led ITE course.
- Episode 2 illustrates the experiences of two PGCE students following the School Direct ITE route.
- Episode 3 turns to the story of two Newly Qualified Teachers towards the end of their first year as qualified teachers.

- Episode 4 analyses the perceptions of one school-based mentor who supports traditional and School Direct ITE students in her school.

My findings from the data are discussed in Chapter 8, which follows these four episodes, including a synthesis of the themes articulated by the participants. This is followed by my critical reflection on the perceptions and disturbances identified in my findings, set in the current context of teacher education in England in Chapter 9. Finally, my conclusion synthesises the study and makes suggestions for further research.

Chapter 2 The Theoretical Landscape of this Study

Preceding this study, comments from many of my students alluded to an approach to teaching that they think that I would advocate, which suggests to me that there is a perceived certainty to my pedagogical approach. This chapter explains the theory that I use in the pedagogical models that I offer to my students and is integrated with the theory that is used to interpret, explain and justify my teaching and my analysis of my students' narratives. The theoretical framework is set in an experience based model using Dewey's theory of experience and education (1938), within which I will draw parallels with both an experience based model of teacher education and secondary mathematics education. This analysis will be linked to the potential for democracy in mathematics classrooms with respect to pupil's learning as well as PGCE student teacher's learning.

In the induction phase of the PGCE, I introduce my students to Askew's model of connectionist, transmission and discoverist mathematics teacher orientations (1997, 2002). In doing so, these orientations provide me and my students with a shared vocabulary to interpret and analyse approaches to teaching mathematics. Hence, Askew's model is a feature of my analysis of this study as well as an interpretive framework for learning to teach in my university classroom. In parallel to this, my students are exposed to Skemp's theory of instrumental and relational understanding in mathematics (1976). Occasionally students interpret these two frameworks as opposite poles, with connectionist teaching positioned against transmission teaching and relational understanding positioned against instrumental, but teaching and learning to teach are far more complicated than this opposition suggests. The theories articulated by Skemp and Askew provide a framework for analysis and interpretation and are not intended as a *manifesto* advocating that mathematics teachers should teach with a connectionist orientation to facilitate relational understanding.

I also teach my students to use Skemp's interpretation of Piaget's theory of assimilation and accommodation of knowledge into schema (1976, 1993) to interpret how mathematics is conceived and misconceived in their classrooms. Alongside this, I use Bruner's enactive-iconic-symbolic representations of understanding (1966, 2006b) to interpret how the pupils learn in their classrooms as well as to understand how their own mathematical pedagogical understanding develops as teachers' translate their typically symbolic understanding to other mathematical representations encountered in classrooms. As with the theorists mentioned previously, these frameworks are used to provide a shared vocabulary for interpreting the mathematics classrooms that my students observe while learning to teach, as well as interpreting and understanding the teaching and learning that takes place in their own classrooms. Meanwhile, I use these theoretical frameworks to interpret and understand the influence of my teaching on my students' behaviour and beliefs.

2.1 Experience and Mathematics Education

The central theoretical framework for this study is my interpretation of Dewey's theory of experience and education (1938). Dewey's work is concerned with democracy in education and so, it is an appropriate framework to use to interpret secondary mathematics education and secondary mathematics ITE practices that liberate or supress pupils' and student teachers' learning.

Potentially, there is a dualism that exists for student mathematics teachers; on the one hand the empiricism of school-based experiences that students may find immediately useful in their classrooms and on the other, the rationalism of predominantly theoretical university-based education that students may reduce to intangible interests of university lecturers, lacking application to their classroom context. To analyse this dualism I will apply the educational philosophy and theories of Dewey to my analysis of research into the PGCE secondary mathematics student teachers' beliefs and experiences. Matheson (2015) argues that Dewey's influence on a constructivist view of learning is consistent with the empiricist view of epistemology because knowledge is derived primarily from experience of the world in which knowledge is sited. However, Dewey described himself as a pragmatist, whereby, as Peters claims, he *"puts himself forward as a theorist of inquiry rather than a theorist of knowledge"* (2010, p. *2*), citing Dewey's claims that knowledge is socially constructed or a 'funded experience' from which all involved in the experience may learn. In this respect, pupils and teachers contribute to learning and the learning is situated within the culture in which the experience takes place. Dewey's conception of knowledge lies in the domain of the sensory experience, which marries with the empiricist view, but is reproduced in actions within the culture in which the knowledge is situated and therefore aligning more closely to a pragmatist epistemology (Peters, 2010), where learning is the product of the interaction between the learner and the environment. Dewey argued that for learning to occur democratically, the interaction must be grounded in the experience of the learner and not the teacher's conception of what the learner's experience should be. Dewey did not disregard the place of rationalist, reasoned argument in mathematics education, but argued that the origins of logical abstractions that contribute to mathematical thinking lie in the logic of experience.

Peters (2010) argues that Dewey and James are credited with making major contributions to the philosophical tradition of pragmatism, most commonly associated with the anti-Cartesian philosophy of Peirce. Peters describes how Dewey's version of pragmatism was distinctive because *"He combined James' emphasis on the acquisition of knowledge as an active and exploratory process, rather than a kind of passive contemplation, with the view, present in much pragmatism […that…] instruments of thought are human constructions."* (2010, p. 2) From this perspective, Dewey proposed that pragmatism combines theory and practice to support intelligent practice, as opposed to indiscriminating practice that may be socially or culturally reproduced through 'traditional' models of education. Thus, the place of Dewey's theory in this study is relevant to my analysis of the 'funded experience' of student teachers as they navigate the learning landscape in school (in classrooms or elsewhere), university (in my classroom or elsewhere) and beyond. Simultaneously, their 'funded experience' in university mathematics teacher education will incorporate a model of mathematics teaching for secondary classrooms that fosters the active participation of pupils, rather than the passive receipt of the teachers' preconceived knowledge.

Dewey's earliest work is over a century old, but, as Matheson (2015) has pointed out, his work has had an influence on the evolution of constructivist interpretations of teaching and learning. Constructivist interpretations of learning dominate contemporary mathematics education research (Lerman, 2014) which suggests that much of Dewey's writing would resonate with issues relating to democracy, which arise in the current education context discussed in relation to social and cultural factors here, as well as the political context of mathematics education discussed later.

Undoubtedly, my own epistemological assumptions that guide my beliefs about teaching and my research contrast with some of the students’ and school-based mentors’ beliefs included in this study. Accounts of the impact of a culture of performativity within our current education system drenched in the political influences of globalisation and neoliberalism are well documented (Ball, 2003, 2006; Furlong, 2005; 2013). The influence of this culture on mathematics classrooms is palpable to student teachers presented with an examination driven model in school-based learning that may be perceived to be in conflict with the experience model presented in university mathematics teacher education (Walshaw, 2010; Brown & McNamara, 2011). In light of this political context, experience based models of education are set in direct conflict with a culture that is centred on delivering curriculum content in a context that is constrained by time and external measures of school effectiveness through examination performance. This political intervention evolves into an epistemological regime, implicit in schools, which is critiqued in my university classroom. The constructivist principles that underlie Dewey’s experience model sit in conflict with a culture of performativity. Similarly, Dewey’s concept of teachers’ psychologising the subject matter, conflicts with the examination driven school context experienced by many of my students, which will be discussed later in this chapter.

Despite the conflict described above, applications of Dewey’s philosophy and theories to mathematics education are numerous. In contemporary classrooms, these applications provide a position for interpreting teacher's beliefs about the purpose and practice of education. I will derive this application from my interpretation of Dewey's writing in *Democracy and Education* (1916) and *Experience and Education (1938)*. From the perspective of students' learning, Dewey's theories interpreted from *How We Think (1933)*, provide a framework for analysing aspects of learning that incorporate reflection, relationships, communication, selective experience and concrete experience within schools (Mason & Johnston-Wilder, 2004). It was through Dewey's belief that education is a social process that he developed a philosophy of education based upon experience and social interaction. Dewey argued for a curriculum that is not constrained by content knowledge delivered to inactive students, but rather an approach that centres the child in the education experience, allowing the child to build on his or her existing knowledge and experience in order to gain command of his or her full learning capacity (1902, 1916, 1938). To this end, Dewey saw schools as social institutions where social reform could take place (1916).

Dewey (1916) introduced a philosophy of education and theory of knowledge that resonates with many issues in constructivist learning models (Ernest, 1991; Lerman, 1996) and advocated "*creating a wider and better balanced environment than that by which the young would be likely, if left to themselves, to be influenced"* (1916, p. 30). Dewey believed that education serves a social function within the school environment. It is the mathematics teacher who assumes the role of the intermediary within this environment, contributing to a setting in which a pupil is likely to learn because the pupil makes use of their experience in learning mathematical concepts. His earlier work was credited with being the stimulus for child centred or progressive education initiatives popular at the start of the twentieth century, but he was critical of what this educational development became. Mason and Johnston-Wilder summarise this as follows:

> [He] developed an approach to learning that was based on the experience of the learner, and on learners actively making use of their own 'powers' to explore and make sense of the world. The much maligned 'discovery learning' and 'child-centred education' were derived from the approach, but being oversimplified, soon turned into the opposite of what Dewey intended.
> (2004: p. 43)

Dewey's approach did not propose to remove the teacher's influence from the learning process, but placed the teacher as the intermediary between the environment where learning takes place and the learning constructed through active involvement. By revisiting his earlier theories in Experience and Education, Dewey sought to clarify his philosophy of experience in order to address criticisms of his earlier work (1938). Dewey defended his philosophy of experience because "*it does not follow that progressive education is a matter of planless improvisation."* (1938, p.28) He recognised the limitation of "planless improvisation" characteristic of discovery learning, as much as the limitations of the transmission orientation of 'traditional' education. This theme is consistent with the findings of Swan (2005, 2014) and Askew (2002), as articulated in their criticism of the discoverist teacher orientation, which proved as ineffective in raising attainment as the transmission teacher orientation. These teacher orientations will be discussed later.

The social context of learning is a key feature of Dewey's education theory. May and Powell (2008) argue that social scientists should concern themselves with the social contexts in which actions and experiences take place. They argue that "*For Dewey ideas and actions* *should be judged according to the social situations which give rise to them*" (2008: p. 41). From this, Dewey identified the need for education to stimulate a child's powers through recognisable experience within social situations in school (Dewey, 1897). Within the experience model, learning becomes a shared enterprise between the teacher and the pupils, and not an exercise in the teacher transmitting knowledge that passive learners are intended to receive (Berding, 1997). Berding describes Dewey's interpretation of the role of the educator whereby "*education is a matter of finding a balance between freedom and control, and between the child as an individual and as a social being."* (1997, p. 28) Within this articulation lies a fundamental issue apparent to beginning mathematics teachers, namely the difficulties associated with finding a skilful balance between freedom and control in classrooms where their control is limited and their sense of agency is restricted by borrowing their classrooms from their school-based mentors.

The role of the teacher is expanded further in Dewey's account of the reciprocal relationship between teacher and student:

> *The plan […] is a co-operative enterprise, not a dictation. The teacher's suggestion is not a mold for a cast-iron result but is a starting point to be developed into a plan through contributions from the experience of all engaged in the learning process. The development occurs through reciprocal give-and-take…*
>
> (1938, p. 73)

These words contrast directly with Dewey's description of traditional education that lacks co-operation and reciprocal *give-and-take*, where knowledge is transmitted to passive recipients in a manner that has been conceived by the teacher. Learners are not called upon to actively construct knowledge, but to replicate the knowledge as it is transmitted by the teacher's dictation. As Dewey asserts, an experience model of education is not an easy one: *"There is incumbent upon the teacher who links education and actual experience together a more serious and a harder business*" (1938, p. 76). When beginning teachers analyse an experience model of mathematics education, they are critiquing a "more serious and harder business" than that of the teacher who adopts a transmission model. A setting that acknowledges contributions from pupils and teachers, whereby learning involves a degree of reciprocity and a careful balance of freedom and control, is far more daunting to beginning teachers than one which maintains the control of the teacher by transmitting pre-conceived concepts through a linear progression of tasks that have been predetermined by the teacher.

There is a relatively recent trend in England and Wales, that was recommended in government documentation (DCFS, 2008) distributed through national strategies, of stating the objective of a lesson at the start of each lesson, concluding with reference to the lesson objective and identification of success criteria designed to indicate to the pupil and the teacher that the lesson objective has been met (or not). This widespread practice was introduced in response to assessment for learning initiatives that attempted to encapsulate the practices of Black and Wiliam's research into assessment: 'Inside the Black Box' (1998). As with Dewey's early theories, Black and Wiliam's proposals have been subject to misinterpretation, due to the rigidity with which they have been applied. The finding that attainment is raised when the teacher and pupil agree objectives through dialogue and questioning has been translated to a uniform statement of the objective for the whole class.

In 1916, Dewey warned against an emphasis upon a statement of the aim of a lesson, because:

> *If the statement of the aim is taken too seriously by the instructor, as a meaning more than a signal to attention, its probable result is forestalling the pupil's own reaction, relieving him of the responsibility of developing a problem and thus arresting his mental initiative.'*
>
> (p. 91)

Telling students what they will learn fails to recognise the pupils' own powers and experience, ensuring that the teacher is in control of the proposed learning experience. In relieving the pupil of responsibility for developing a problem, the teacher contributes to a culture of dependence on the teacher. If a student teacher tried to change this culture to encourage more resilient problem-solving, it is likely that the pupils would resist this change (Bruner, 1996, 2006a; Johnston-Wilder & Lee, 2010). Starting lessons with pupils copying the learning objective from the board exemplifies the policies and practices that PGCE students are asked to follow during their teaching practices, irrespective of their beliefs about how learning is best fostered in their classrooms. This practice demonstrates that the teaching is not modelled on "planless improvisation" which Dewey guards against, but nor is it characteristic of learning supported through "reciprocal give-and-take" and experiences funded by contributions from the teacher and the pupils.

Dewey proposed that physical or practical experience in the classroom could provide the logical foundations of mathematical thinking in some concepts. Oversimplification of this approach could be criticised for the failure to appreciate the nature of abstract mathematical thinking. Sleeper defends Dewey's position on logical thought:

> *Dewey's point was to make clear that he was not attempting to undermine the structure of higher mathematics, merely that he had worked out his own theory to account for the ultimate origins of abstract concepts. He was saying that if mathematics has any logical foundations, those foundations are to be found in the logic of experience.*
>
> (Sleeper, 1986, p. 60)

Dewey believed that the logic of experience was sensed by students who were actively contributing to the learning process. Yet he was aware that the transition from a concrete experience to a mathematical concept was not an automatic development in the minds of the learners. The experience of measuring a pencil, identifying features of a shape or counting beans provide the *foundations* of mathematical systems and abstractions; they formed the origin of a concept. Dewey cautioned against the use of apparatus, with the aim of aiding learning, without allowing the intended meaning of the apparatus to be realised. He did not assume that an experience model of mathematics education was adopted simply because concrete tools or images are used to represent symbolic concepts. He illustrated this point:

> *Instruction in number is not concrete merely because splints or beams or dots are employed. Whenever the use and bearing of number relations are clearly perceived, a number idea is concrete even if figures alone are used… if the physical things used in teaching number or geography or anything else do not leave the mind illuminated with recognition of a meaning beyond themselves, the instruction that uses them is as abstruse as that which doles out ready made definitions and rules, for it distracts attention from ideas to mere physical excitations.*
>
> (Dewey, 1933, p. 224-5)

Many mathematics educators have echoed Dewey's message (Hart, 1981, Mason, Burton & Stacey 2010). Without appropriate experience to allow learners to make connections with the physical or visual 'aid' to learning and the concept to be learned, there is little to be gained by using the 'aids' to develop understanding of a concept. This is the stage of "*authenticating a part of formal mathematics*" (Hart, 1993, p. 27, quoted in Mason &

Johnston-Wilder, 2004, p. 256) that may be omitted if conceptual understanding is assumed simply by manipulating physical tools or representing ideas in diagrams.

Figures 2.1, 2.2 and 2.3 show some of my students teaching algebra in a manner that is designed to model a meaningful use of symbols in early algebraic generalisations learned by 11-year-olds.

Figure 2.1: Pupils are introduced to a 'balancing puzzle' using sweets.

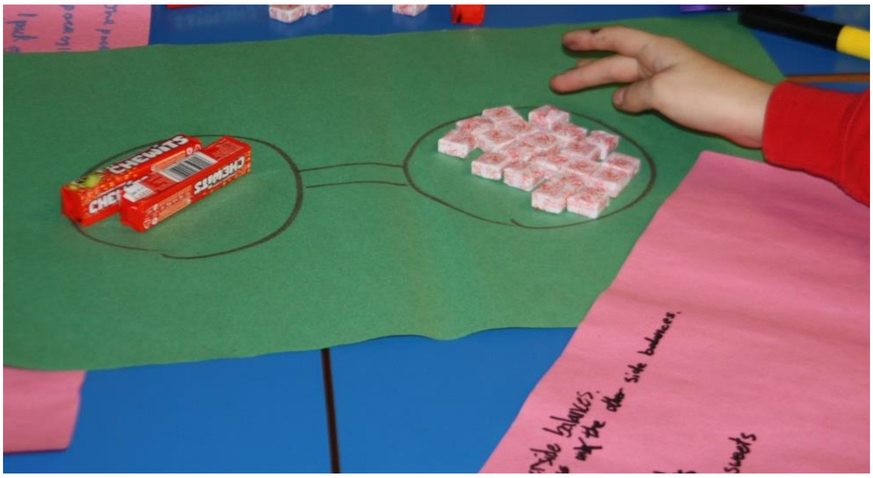

Figure 2.2: Pupils manipulate the problem to find 'the number of sweets in a packet'.

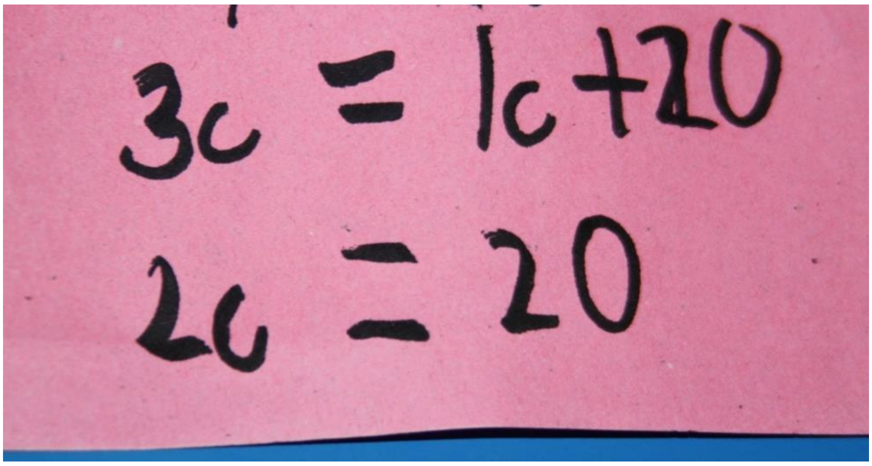

Figure 2.3: *Pupils represent the sweets puzzle symbolically*.

The details of the full series of lessons represented in figures 2.1 to 2.3 are not included here, but Figure 2.1 and 2.2 show that the pupils have been introduced to a balancing problem using a physical model whereby the 'number of sweets in a packet' represents an unknown number. Figure 2.3 shows that this pupil has interpreted this problem using algebraic symbols, where c represents the number of sweets in the packet. This may indicate that this pupil is making the transition from the physical puzzle to successfully constructing and manipulating an equation. Within this, the bottom of Figure 2.2 suggests some notes that the pupil has written in making this transition using phrases such as 'the other side balances'. This image could represent the 'logic of experience' described earlier because the experience of balancing sweets puzzles may leave behind some meaning in the mind of the pupil who is solving linear equations like $3c = c + 20$. Within the logic of the sweets puzzle, it may be clear to the pupil that the number of sweets in two packets is equivalent to twenty sweets. However, in the absence of a physical model, one traditional process of balancing by 'subtracting c from both sides' may mean nothing at all beyond replicating a process demonstrated by the teacher.

Dewey warned that physical activities like the sweets puzzles may be reduced to mere physical excitations because the pupil sees the algebraic equation and the puzzle as unrelated representations of a concept. For the teacher who is not adept at balancing the pupils' freedom and guidance within the lesson, the two representations would remain separate; two separate activities within one lesson where no transition is made between either representation. Similarly, Dewey knew that the activity like the one depicted above is the physical foundation of a concept that has contexts that cannot readily be represented by a physical model. In this respect, Dewey believed that the logical foundations of a concept, founded in experience, would allow the pupil to recognise meaning beyond the physical model. He did not intend to undermine the complexity, precision and elegance of higher mathematics, but determined that mathematics would be accessible to learners whose ideas were founded on the logic of experience.

In *How We Think* Dewey wrote about cognition in terms of generalisation and synthesis (1933), which are two fundamental principles of learning abstract mathematical concepts (Polya, 1957; Mason et al., 2010). He describes attempts to root pupils' education in experience that have failed because connections are not made with the conceptual development that the experience is intended to support:

> *A variety of worthwhile experiences and activities with real materials is introduced, but pains are not taken to make sure that the activities terminate in that which makes them educationally worth-while - namely, achievement of a fairly definite intellectualization of the experience. This intellectualization is the deposit of an idea that is both definite and general […] What does having an experience amount to unless, as it ceases to exist, it leaves behind an increment of meaning, a better understanding of something, a clearer future plan and purpose of action: in short, an idea?*
>
> (Dewey, 1933, p. 154)

Here, the nature of the experience described by Dewey is far more subtle than the mere inclusion of physical representations of mathematical abstractions. For a learner who experiences ‘the number of objects in a packet’ model of early algebraic generalisation, there is no algebraic meaning behind the physical puzzles unless the learner experiences the relationship between the physical and symbolic representations of the problem. When the physical problem ceases to exist, the learner may or may not understand the meaning of algebraic symbols representing a variable quality or unknown value. Pupil’s making meaning from experience in mathematics lessons can have many representations (Bruner, 1966, 1996) and involves teachers embarking on the difficult process of balancing the pupils’ freedom and the teacher’s control within their classrooms. This theme alludes to the complex nature of learning mathematics that will be related to learning environments that stimulate relational understanding of reasoned processes within mathematics concepts in the next section (Skemp, 1993).

2.2 Learning mathematics in the secondary school

Skemp (1993) described school mathematics as non-mathematics, because it is a seemingly meaningless series of processes which undermine mathematical thinking. He developed programmes of primary mathematics education that were based upon his principle of ‘real mathematics’ founded on connections and reasoned processes. It is over forty years since Skemp first published criticisms of school mathematics, nevertheless my current perception of secondary school mathematics is generally similar to Skemp’s, albeit not exclusively. I teach Mathematics Education to PGCE Secondary Mathematics students, beginning teachers navigating their way towards qualified teacher status. In my teaching, I model how mathematics classrooms that support the development of relational understanding in mathematics (Skemp, 1976) could be characterised. Simultaneously, I place my students in schools where they report that much of the mathematics teaching that they observe promotes instrumental understanding. This exemplifies the origin of a tension that influences my role as a teacher educator and inevitably becomes a feature of initial teacher education for my students (Zeichner, 2010; Davey, 2013). PGCE Mathematics students perceive conflicting messages from classroom teachers and university tutors and must somehow make sense of these messages as they develop their own philosophies as mathematics teachers, influencing their professional identities (Raffo & Hall,2006; Williams, 2011) as well as how they behave within a school mathematics classroom (Clandinin et al., 2006; Murray, 2008; Walshaw, 2010; Brown & McNamara, 2011; Stronach, 2011).

In analysing conflicting advice, I will account for the epistemological assumptions that inform my approach to teacher education and undoubtedly inform the pedagogical methods that I model in my teaching. I will link my epistemological assumptions to empiricist and rationalist conceptions of thought and knowledge, using my interpretation of teacher mathematical knowledge (Shulman, 1986, 1987; Jones & Straker, 2006).

Disharmonies are a feature of this study. Opposition exists within two epistemological interpretations of knowledge, rationalism characterised by individual, deductive reasoning contrasted with empiricism (Russell, 2009) which emphasises the role of experience and evidence in the construction of knowledge. Differences in these two systems of thought are linked to opposing interpretations of mathematics teaching and learning (Carr, 2003). Askew (Askew et al., 1997) identified dominant mathematics teacher orientations that include a transmission oriented teacher whose view of mathematics is a fixed body of knowledge comprising skills and procedures that have to be delivered to learners by allowing them to watch, listen and imitate the teacher until mathematics fluency is achieved. Alternatively, a connectionist oriented teacher views mathematics as a body of ideas and reasoning processes that are connected, which positions learning as a collaborative activity in which learners are challenged and explore meaning so that understanding is reached through dialogue between the teacher and the learners (Askew et al. cited in Swan, 2005).

Deductive reasoning dominates the classroom of the transmission teacher, whereby a series of deductions are made, facts are presented so that the truth of the deductions cannot be questioned because they follow the logical structure of mathematics. From a rationalist perspective these mathematical truths do not require or merit sensory experience because one truth is built upon a previous accepted truth (Carr, 2003; Cottingham, 2006). It is sufficient for the learner to measure his or her own mathematical knowledge through the presence or absence of correct answers so that they can get on in the world of secondary school mathematics. Bookwork, tests and examinations signify success or failure of mathematics learning in secondary schools, so that the student can or cannot ‘do maths’, regardless of whether they are capable of mathematical thinking, logical reasoning, exercising spatial awareness or applying mathematical thinking in a variety of contexts. Often, the latter aspects are beyond the secondary school learner’s perception of mathematics (Nunes & Bryant, 1997). The rational subject, the secondary school learner, needs only the confirmation of test results to gain an identity as a mathematics learner within the symbolic order of the secondary school (Oliver,2002). The transmission teacher builds on *a priori* knowledge (Carr, 2003, Cottingham, 2006) so that experiment and experience are unnecessary to the learner as he or she builds *secondary school* mathematical knowledge (Askew et al., 1997).

In contrast, aspects of Askew's articulation of a connectionist teacher can be viewed from an empiricist perspective (Carr, 2003; Sorell, 2006), taking the interpretation of mathematical knowledge developed from experience and evidence. An experience in the classroom facilitates learning because the teacher presents mathematics in a manner that allows the learners to build schema through meaningful connections (Skemp 1993, Swan 2005). Mathematical knowledge is socially constructed through scaffolded experiences. Reason is built from dialogue and experience. However, Lave (1996) argues that the nature of situated learning is distinct from traditional conceptions of learning inherent in the rationalist or empiricist views of learning because:

> *Theories of situated activity do not separate action, thought, feeling, and values and their collective cultural-historical forms of located, interested, conflictual, meaningful activity. Traditional cognitive theory is "distanced from experience" and divides the learning mind from the world.*
>
> (Lave, 1996, p. 7)

In this respect, Lave's interpretation of learning recognises the cultural influences on the learner at the location of learning as well as the influence of past experience, beliefs and emotions that characterise the learner. In mathematics classrooms learning is far more than the cognitive response of pupils; it is the product of social, cultural and historical influences at play in the classroom and beyond (Lave, 1988; Wenger, McDermott & Snyder, 2002). This shifts conceptions of learning mathematics away from knowledge acquisition, towards versions of socially constructed knowledge that are subject to the influence of past experiences and current actions of people acting together in the classroom (Lave, 1996). Despite Lave's distinction between cognitive and social theories of learning, there are parallels that are important in this study because in each perspective, it is not that learning occurs that leads to difficulty for beginning teachers, but it is what is learned that is problematic.

Thus, in mathematics education the distinction between transmission and connectionist teacher orientations is more complicated than to assign each to rational or empirical systems of thought, because learning is arbitrary in the context in which the learning is situated. In mathematics learning it is impossible to argue for an absence of *a priori* knowledge because, as Skemp argued in his interpretation of Piaget's accommodation and assimilation theory (1993), schemas are built by expanding, adjusting or accommodating novel ideas into existing structures of knowledge. This is not *a posteriori* knowledge in the true empiricist sense because *a posteriori* knowledge depends on experience and empirical evidence. Mathematics lies outside the natural sciences because it is built upon agreed truths and not probable truths observed from experiment or observation. The connectionist mathematics teacher does not propose to reject the agreed truths in favour of empirical evidence and experience, but attempts to create an experience where a learner can understand the agreed truths of mathematics through sensory experience and dialogue. This shared or 'funded' experience interpretation has more in common with Dewey's model of experience based education than the model of the 'traditional' transmission classroom.

The association between an empiricist system of thought and mathematics learning comes from how knowledge is constructed and not the derivation of the knowledge. In this sense, when I expose my students to a connectionist model of teaching, I am offering a constructivist model of education, but one that attempts to build meaning in mathematics through the acknowledgement of a cognitivist conception of learning (Skemp, 1993; Swan, 2005). My student teachers experience models of learning mathematics that expose rather than correct misconceptions. From a cognitivist perspective, if a misconception has been assimilated into a pupil's existing structure of knowledge, then the teacher needs to involve the pupils in an experience that conflicts with the misconception. The subsequent disturbance leads the learner to seek new information so that the concept can be accommodated into the students' structure of knowledge, and equilibrium in learning is restored. Lerman (1996) describes this interpretation of a cognitivist view of learning as radical constructivism derived from Paiget's theory of assimilation and accommodation, contrasting with social constructivism that does not separate the individual's cognition from the social situation in which the cognition arises. Using Lerman's interpretation, the connectionist teacher orientation has traits in common with both radical and social constructivism, but the passive receipt of knowledge characterising the transmission orientated teacher has traits of neither.

Whilst I am confident that that this is an ethnographic study, my research does not sit exclusively within one epistemological framework. There are similarities between empiricist and pragmatist perspectives seen in the situated nature of knowledge, constructed through experience and social interaction, which also echo models of learning within social constructivism. Within this picture I also use a view of knowledge aligned to cognitivist or radical constructivist principles (Lerman, 2014). It may be that this study does not readily align to one paradigm because the study exposes the complicated, situated nature of learning to teach. This presents a potential discontinuity in the phenomenon of learning to teach because policy reforms position the teacher as a rational being more closely to relativist principles (Furlong, 2013) and do not recognise the social, cultural, emotional and historical influences on teachers' professional learning.

The connectionist and transmission orientations provide contrasting conceptions of mathematics pedagogy that echo the dialectic between instrumental and relational understanding in mathematics (Skemp, 1993). Mathematics teaching that attempts to build instrumental understanding presents concepts as a disparate body of facts, processes and procedures that must be learned separately and replicated exactly as they are presented by the teacher. When viewed from a relativist perspective, when building instrumental understanding the mind makes mathematical meaning in the absence of sensory experience; the thinking mind is separated from the physical body (Rata, 2012). There is an absence of disturbance for the learner because the series of deductions are transmitted by the teacher in a way that limits disequilibrium. Difficulty is countered by a repetition of the transmitted process, by being 'shown again'. On the other hand, the teacher that attempts to build relational understanding is more likely to complement the view of the connectionist oriented teacher, that mathematics represents a connected body of ideas and concepts that can be learned through inquiry, dialogue and sensory experience. Instrumental understanding may be underpinned by blurred reasoning, but is accepted because it registers the approval of the teacher or examiner, yet relational understanding has more distinct significance because it is reasoned with existing schema so that sense can be made of mathematical concepts with or without the teacher's approval. Pupils are not expected to remember, but to understand. Within this, pupils can experience the disturbance of not knowing in the search of knowledge that allows schema to be expanded, knowledge to be accommodated in the pursuit of equilibrium (Skemp, 1976; 1993). Relational understanding is built on knowledge and reason, in most cases reasoning from existing knowledge by building on key known facts (Watson, Jones & Pratt, 2013). For these reasons, calling on beginning teachers to develop relational understanding in their classrooms is asking them to embark on a more difficult undertaking than to build a classroom culture based on instrumental understanding. Relational understanding may be developed through dialogue and reciprocal exchange between students and teachers, but in mathematics, it may also be developed though other signs, such as the ability to predict an outcome successfully, even though the ability to articulate how the predication was made may lag behind the ability to make sense of the signs and symbols that are understood as mathematics (Mason, Graham & Johnston-Wilder 2005; Mason et al., 2010).

Although this last statement is a simplification of the complexities of learning mathematics it is an important feature of this study in order to distinguish some of the principles of a constructivist mathematics classroom from that of other disciplines. There are a number of features of my interpretation of mathematics learning that are not wholly aligned with either relational or instrumental understanding. In this respect, my interpretation of mathematics learning is not solely articulated within descriptions of a connectionist or a transmission teacher orientation. Rata cites Moore and Muller's interpretation of the social construction of knowledge that fails to separate the knowledge from the knower:

> *Only within that social construction of the recognised image of who we are meant to be, can the human exist fully as a social being according to this approach. Knowledge can be nothing more than the construction of the knower; the product of social groups and their interests (Moore & Muller, 2010).*
>
> (2012, p. 117)

Knowledge viewed from the position of social construction can be applied to mathematics teacher education as well as mathematics education in the classroom. From the perspective of the secondary classroom, the way in which a pupil makes sense of a mathematical 'agreed truth' is subject to the conditions in which the pupil is exposed to the agreed truth; who the pupil is with, how they are inclined to view mathematics, the pupil's emotional response to the experience, the nature of the experience in that time and place, as well as a whole host of other ad-hoc factors. The accuracy of a mathematical truth is not in question, but the manner in which the pupil constructs their knowledge of that truth is subject to the way in which the pupil identifies with the experience encountered.

As with all secondary school subjects, the manner in which concepts are constructed in the mind of the learner is subjective (Bruner, 1996, 2006b), related to the experiences and dispositions of the individual learner and the culture of learning. However, secondary school mathematics is based on agreed truths, unlike, for example, the probable truths encountered in science and humanities. Building a structure of knowledge in the secondary school mathematics classroom has inherent complications because of the host of ways in which pupils have learned (or not learned) existing key facts. Progression in secondary mathematics, as in other subjects, is not linear, but connections in the structure of knowledge within and between concepts create enormous challenges for beginning teachers who are tasked with creating learning environments that allow pupils to build on their existing knowledge. There is an absence of research that directly compares the pedagogy of secondary mathematics to other subjects, or the pedagogical content knowledge (Shulman, 1985; Krauss et al., 2008) of beginning English secondary teachers of mathematics to other subjects. However, anecdotal narratives from teachers who have crossed disciplines suggest that variation in the structure of pupils' existing mathematical knowledge presents a prevailing issue in mathematics classrooms.

A similar, but not identical, argument can be applied to the mathematics pedagogy learned by the PGCE student. In the case of mathematics pedagogy the truth of the knowledge developed is subjective. This serves to further complicate the nature of learning to teach mathematics because of the absence of agreed pedagogical truths. The similarity lies in the social construction of the teachers' pedagogical knowledge positioned as the product of the social groups and the interests of the social groups that the student teachers encounter in navigating the learning landscape (Wenger, 1998). Brown describes the nature of cultural renewal encountered in the transition from student of mathematics to student of mathematics education:

> *The students were bringing together their past knowledge from school with new observations centred in their current task of training to teach to produce new understandings. Their conceptions of mathematics are being shaped to new circumstances. Individually they are moving from being school students to trainee teachers. But in doing this they are also adapting to new collective understandings of what it is to teach mathematics in schools. In school, as pupils, they were sharing that generation's absorption and construction of school mathematics. In college as trainee teachers they are participating in a*

cultural renewal of mathematics since the schools in which they will teach will collectively influence what mathematics is for the next generation.

(Brown, T., 2011, p. 50)

The nature of the cultural renewal that the student teachers experience is subject to influences from all parties involved in their teacher education, whether that is their university or school educators or anyone else involved in the construction of the student teachers' perceptions of school mathematics. Student teachers will choose to perceive mathematics, consciously or subconsciously, as a function of the influences upon them at a particular time. In the case of this study, dominant influences come from their encounters located in university-based or school-based mathematics teacher education. This study will capture the student teacher's articulation of their perceptions of mathematics and mathematics pedagogy and will interpret how they behave in the secondary school classroom in relation to this articulation.

2.3 Representations in a Structure of Knowledge

I have described an experience based model of mathematics education without defining how knowledge is learned within that experience. Dewey invented the term, "psychologising the subject matter" (1933, p.14) to describe the way in which a teacher transforms that which is to be learned into an experience or representation that is learnable. In doing so, the teacher organises experiences that motivate the pupils to tackle problems because the pupils can relate to the experience integral to the problem. Bruner proposed enactive, iconic and symbolic (EIS) representations that characterise a domain of knowledge (1966) and I use these representations to teach my student teachers different models for learning mathematics that could contribute to experiences that present mathematics to pupils in a learnable manner. In other words, enactive, iconic and symbolic representations provide a framework for guiding my students to 'psychologise the subject matter'.

A basic reduction of the EIS representations is actions, pictures and symbols. Enactive representations of learning incorporate active involvement in physical activity. The physical or practical activity constitutes a "*set of actions appropriate for achieving certain results*" (Bruner, 1966, p. 44). The iconic representation is the development of a set of images designed to assist the pupil in imagining a concept, without fully defining it. The learner is able to imagine a concept without activity and to represent the concept using diagrams, images, words or informal jottings. The symbolic representation is a "*set of symbolic or logical propositions drawn from a symbolic system that is ruled by laws for forming and transforming propositions"* (Bruner, 1966, p. 44). The symbolic representation results in an idea, a formalised concept. When my students use the EIS framework in their practitioner enquiries, they often successfully apply the framework to classroom interventions designed to develop pupils' geometrical thinking. These students seem to gain a clear understanding of the need for action to allow their pupils to learn what they cannot initially say or draw. Rotation without turning, congruence without laying one object over another are potential obstacles to their pupils' learning and so these activities are incorporated into my students' lessons more readily. In teaching geometry, some students seem to be open to enactive representations of mathematical concepts. However, they often learn how the transformation from an enactive experience to a symbolic mathematical concept is problematic.

Bruner proposed that the transition from enactive to iconic to symbolic understanding was neither a smooth nor linear process because "how the nervous system converts a sequence of responses into an image or schema is simply not understood" (Bruner, 1966, p. 14), but he did acknowledge that doing and imagining results in a formalised generalisation through shifts between representations. His earlier work suggests that he believed that learners make the transition from enactive to iconic and from these two representations to symbolic in learning a concept. However, his later writing on culture in education acknowledged that this may not be the case when the situated nature of learning is considered (Bruner, 1996, 2006b). I have taught my students to use the EIS framework as a progression of representations to guide their planning and support for learning when teaching concepts that are believed to be new to their pupils. However, most concepts taught at secondary school are not wholly new and pupils bring with them arbitrary structures of knowledge that relate to the concepts met in each lesson. This leads to difficulties for teachers when they are presenting problems to pupils in one representation, but the pupil's learning is affected in another. With this in mind, I teach them to consider when to present a new concept through enactive problems and when to use rich problems to help them to diagnose what their pupils know about a concept and how they make sense of the relations within the concept. In order to support their pupils, I teach them to be aware of enactive and iconic representations and to create a climate where pupils have the freedom to move between representations, until symbolic understanding is achieved.

Like Bruner, I believe that in most cases, pupils learn by moving from enactive and iconic representations to symbolic because pupils' ideas, formalised concepts, are created through activity that provides a concrete experience and a way of imaging and imagining what ultimately results in an idea, a learned concept. Like Dewey, I believe that we are narrative creatures, so that the enactive and iconic experiences that pupils enjoy, give them a means of telling the story that is eventually a mathematical concept. The pupils reduce unnecessary information in the story until they reach the point where they can narrate a generalised, formalised concept. The symbols 4^3 are nothing more than ink on paper without stories involving 'three fours multiplied together' or the 'product of three fours' or physically representing the 64 centimetre cubes in a four centimetre cube, appreciating four layers of sixteen cubes. It is these beliefs that reinforce my behaviour in my university classroom and, hence, strengthen my certainty that I want my students to critique approaches like the ones described above alongside their reflections on their models experienced in school.

Bruner knew that actions, pictures and symbols vary in difficulty and efficacy for people depending on their age, disposition or personal history (1966, 1996). This exposes one of the main hurdles to developing my students' pedagogical mathematical knowledge (Shulman, 1987). All of my students start the PGCE course having demonstrated necessary mathematical knowledge through their qualifications and in some cases through subject knowledge enhancement courses, preceding the PGCE. Most of them are adept at telling the stories of mathematical concepts through symbolic representations. However, the pedagogical model that I teach them requires them to expand their symbolic understanding, or possibly undo it, so that they can make sense of enactive and iconic representations.

Within the transmission teacher orientation in classrooms characterised by the pursuit of instrumental understanding, symbolic representations of mathematics dominate. Where iconic representations exist, they are presented as the recurrent habits of the teacher and not the summary images of pupils on their way to finding a more comprehensive definition of a concept. During lessons in the transmission classroom, synonymous with Dewey's 'traditional' classroom (1938), the teacher presents definitions of the symbolic mathematical representation of a concept and provides pupils with explanations and examples to support the agreed truth of the definition. In the classroom where the teacher creates stimuli to support learning by psychologising the subject matter, this could also include mathematics represented symbolically, but the representation is inherent in a tantalising problem that the pupil can relate to. In the experience based model, the teacher would know how to transform the subject matter into a representation that is synchronised with the learner by making contact with the experience of the learner (Dewey, 1933, 1938). The teacher would know how to 'psychologise' mathematical concepts into enactive, iconic or symbolic representations because the teacher stimulates the pupils' powers through reciprocal dialogue between the teacher and the learner that allows the child's instincts and powers to be facilitated (Dewey, 1897, 1916).

2.4 Prescribed Curriculum

Currently, there is a great deal of interest in the approaches to mathematics education adopted by places like Singapore, Shanghai, Finland and Korea because they lead world rankings in The Organisation for Economic Co-operation and Development (OECD) measures of pupils' mathematics performance illustrated through publishing outcomes from the

Programme for International Student Assessment (PISA) (Darling-Hammond, 2010). The National Centre for Excellence in Teaching Mathematics (NCETM) currently supervises DfE funded projects through a series of 34 'Maths Hubs' established in Teaching School alliances throughout England. Most recently, these developments include a knowledge exchange programme between primary school teachers from England and Shanghai as well as the introduction of a mathematics textbook based upon the model used in Singapore. In all of the higher performing regions, mathematics education programmes are characterised by many of the principles of a mastery learning model (Bloom, 1971), whereby what is to be learned is organised into units, with the time allocated dictated by the pupils' understanding or mastery of the concepts within each unit and not the length of time allocated on a school's scheme of work or learning programme (Darling-Hammond, 2006, 2010). In each case, these countries' approaches to mathematics education share many of the characteristics of relational understanding, built through problem solving that constructs knowledge based on pupils' existing understanding using enduring learning models and subtle variations that facilitate generalisations and connections (Watson & Mason, 2006). In scrutinising each country's approach, I can interpret learning models using the theories of Bruner, Skemp, Piaget and Dewey that I have described earlier as well as other influences such as Gattegno (1974) and Mason (2010). The irony that England is currently importing approaches to mathematics education that evolved through research in America and Britain is not lost on a number of observers in associations that I am affiliated to (Hodgen, Monaghan, Shen & Staneff, 2014). Although the emergence of the activities of the NCETM's Shanghai-England teacher exchange are too recent to ascertain their exact theoretical foundations, my use of British mathematics education periodicals and refereed journals exposes the prevalence of the influence of the principles of variation and mastery in the mathematics pedagogies communicated by university teacher educators and education researchers nationally (Watson & Mason, 2006). It appears to me that the DfE are turning to examples of mathematics education from South East Asia to apply professional knowledge that is already situated within university-based teacher education in England.

I taught secondary mathematics in South East Asia for six years. It took only a few weeks of teaching pupils from South Korea, Singapore, China and Japan to realise that the popular notion that South East Asian mathematical success being attributed to rote learning was misconceived. The pupils who I taught who had been educated in primary schools in South Korea, were amongst some of the most strategic, fluent mathematicians that I had ever come across. I was fascinated by the origins of their mathematical thinking and frequently watched with awe when succinct, confident solutions to problems were presented to me. Clearly, I had a lot to learn from my pupils' experiences of mathematics education, just as these pupils had a lot to learn from my own culturally reproduced version of mathematics. I welcome the exchange of knowledge between cultures whilst at the same time recognising that the experiences and beliefs that characterise versions of mathematics education differ from country to country as much as they do from classroom to classroom.

The DfE are disseminating the principles of mastery learning (Bloom, 1974; Watson & Mason, 2006; Darling-Hammond, 2010) through a national curriculum for mathematics (2013), illustrated through extracts such as:

> Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems […] The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace.
>
> (DfE, 2013, para 3)

Although there is an absence of direct reference to the theoretical underpinnings of these education models in DfE documentation, I am able to identify phrases that resonate with the theories used in this study. Multiple representations in an interconnected subject is at the heart of Askew's findings within the connectionist teacher orientation. Making connections, reasoning, solving problems and the subsequent fluency that this kindles are at the core of Skemp's articulation of relational understanding in mathematics. Moving fluently between representations resonates with Bruner's Enactive-Iconic-Symbolic representation of understanding. Most pupils moving through programmes together is the backbone of Dewey's democratic model of learning, provided that these programmes are structured through an experience model of education. Interpreting this version of the national curriculum for mathematics (2013) through theoretical lenses is easy to do.

Therefore, at least in terms of the written policy, the theoretical interpretation of pedagogies that drive my university-led teacher education should be compatible with the National Curriculum currently implemented in secondary schools. Yet, compatibility between my teaching and government policy is not a fresh phenomenon because I use English policies, reviews, recommendations and inspections from at least the last 40 years to exemplify theoretical interpretations of aspects of mathematics secondary education that are, at least in part, built on epistemological and pedagogical principles that are compatible with my own. Discontinuities seem to arise in relation to how developments in national interventions into local curricular developments are lived out; how they are contextualised in the schools in which my students learn to teach.

2.5 Disturbances and Discontinuities

These accounts of the theory that inform my pedagogy as a teacher educator, as well as the pedagogy that I ask my students to critique in relation to the secondary classroom, expose potential disturbances for my students. The experience based model that informs my teaching about secondary mathematics education should also, if I am to act on my beliefs about learning, be the pillar of my teaching in the PGCE course. However, my teaching is situated away from the classrooms in which my students are learning to teach. If I want my students to learn their pedagogy through an experience based model, I need to provide them with experiences that will influence their beliefs about pedagogy. In my university classroom I provide students with opportunities to expand their pedagogical mathematical knowledge by solving mathematics problems through enactive, iconic and symbolic representations of concepts; they experience these representations as mathematical models situated in the university. Observers suggest (Ofsted 2008, 2012; Askew, 2002; Swan, 2014) that their dominant experience in secondary classrooms is characterised by transmission teaching, which means that teaching moves quickly towards symbolic representations of mathematical concepts, and that this symbolic representation is built upon an assumption of instrumental understanding. This is supported by countless accounts describing similar experiences from my students. Thus, discontinuities are inherent in an initial teacher education course that integrates school-based and university-based education.

I have already suggested that I teach my students a research informed and theoretically interpreted model of mathematics teacher education. Meanwhile, they learn to teach mathematics in classrooms that they borrow from teachers and mentors in schools for relatively short periods of time, becoming drenched in the culture and context that characterises each school. My analysis of disturbances in relation to these two sites uses Kristeva's notion of abjecting that which upsets the symbolic order of the worlds of university and school (Oliver, 2002). Kristeva uses Lacan's theory of symbolic order to distinguish the communication, relations and conventions that conform within the social world and in the case of this study, the social world of school and university teacher education. Notions of pedagogy that upset the social, emotional and physical order of each site of learning are potentially traumatic for those who operate within the symbolic order. This causes the subjects within the symbolic order to abject notions of pedagogy that do not reflect the version of themselves that they recognise within the symbolic order of their respective sites of learning. This does not mean that notions of pedagogy that are abjected are not recognised by those who abject them, but they are not recognised within the symbolic order of each site of learning. In my teaching, I visit the schools where my students teach, but I am not part of that symbolic order because I do not conform to the conventions within the school culture. At university, I teach mathematics education in a site that is characterised by wider aspects of teacher education and so I am part of the symbolic order of university teacher education, but I do not wholly signify university teacher education. My aim in this study is to research mathematics teacher education, but in doing so, the data exposes many aspects of my students' teacher education. Thus whilst the primary aim of this research is to examine mathematics teacher education, aspects of general teacher education and wider professional knowledge are present in my analysis of my data.

The question of how potential discontinuities impact upon student teachers' beliefs and behaviours in secondary classrooms is the subject of my analysis of the data in later chapters. The following chapter identifies the methodological considerations that I have made in designing this study, together with my justification of reflexivity and narrative analysis in this ethnography.

Chapter 3 Methodological Considerations

In this study I ask six beginning teachers how they learn to teach secondary mathematics. In doing so, the study exposes and explores metaphors articulated by the students as they navigate the landscape of teacher education. Metaphors, offered by the student teachers, form the backbone of their narratives, interrogating their beliefs about learning secondary mathematics as well as their interpretation of how their behaviour is influenced by teacher educators in school and at university. I did not set out to use metaphor to explore my students' narratives in this study and I could not have predicted the phenomenon. Yet, my students' metaphors have become more than a linguistic device, they allow us to communicate our understanding of learning to teach mathematics because the metaphors allow us to cross different domains of learning (Lakoff & Johnson, 2003).

I have been involved in secondary school mathematics education for over twenty years, either as a novice then experienced classroom teacher, a school-based mentor and curriculum leader or most recently, as a mathematics teacher educator. Alongside this, I am now a beginning researcher of mathematics education and mathematics teacher education. Outwardly, I have lived many of the experiences that influence mathematics ITE students. However, I am aware that everything that I have learned about mathematics education has been situated in the institutions and communities in which I have taught (Lave, 1996; Brown, J. S., 1988). I cannot claim to know what it is like to be one of the beginning teachers in this study or one of the school-based mentors described by my students simply because I have lived through both of these roles, but I can acknowledge that my beliefs about mathematics education have been influenced by all of these experiences. This leads me to believe that I can interpret how my students behave and how they articulate their perceptions of their early teacher education because I understand the situated nature of teacher education, the intricate contexts experienced in schools and the potential disturbances apparent in learning to make mathematics learnable. Furthermore, my experience allows me to interpret the perceptions of my students as they learn to teach because I expand my knowledge of them as individuals from the moment they submit an application to their ITE course to at least the point where they become Newly Qualified Teachers (NQTs). This is my position as the researcher in this study and it enables me to account for my students' perceptions as they navigate the landscape of learning to teach mathematics because I am part of this landscape and I understand the terrain (Wenger, McDermott & Snyder, 2002; Wenger, 2013). Thus, my study relies upon reflexivity as a methodological tool to tell my students' stories (Pillow, 2003).

I recognise that student's perceptions cannot be researched through quantitative methods. I have no hypothesis to test or impact to measure, by seeking correlations and patterns that may lead me to a theory about student teacher's perceptions of their own teaching. However, there are data relevant to my research questions that are quantitative. For example, age and prior experience are important factors that contribute to the identification of 'mature' student teachers, together with patterns of career development identified for 'mature' student teachers. Connolly advocates the need to position education research between scientific and interpretive methods through the "*Need to move effortlessly between qualitative and quantitative methods*" (Connolly, 2009) together with "*The importance of being critically reflexive and theoretically informed as we move between* *methods*" (Connolly, 2009). Connolly's arguments echo notions of false dichotomies discussed by Clough and Nutbrown where they cite Carr's argument that distinct and separated paradigms are inappropriate for research in education (Clough & Nutbrown, 2007). These positions are justified by the need to determine what is most relevant to any study, and to which stage of the study particular approaches apply. My approach in this study is predominantly interpretive, recognising that teachers' perceptions are both subjective and unique, thus requiring a small scale qualitative approach to answering my research question (Cohen, Manion & Morrison, 2007; Hammersley & Atkinson, 2007). Similarly, ethnography places me, the researcher, as a participant in the study (Cohen et al., 2007; Crang & Cook, 2007). I am unquestionably part of my study as the interviewer, observer and teacher of my students and so I use ethnography to research my students' behaviour and beliefs about mathematics education. Ethnographic research allows me to interrogate and answer my research questions because I am studying the behaviour, thoughts and perceptions of people within a culture; beginning teachers and the teachers who influence them within the culture of university and school teacher education. I adopt methods that are primarily qualitative in order to identify, understand and interpret the perceptions of my students (Sparkes, 1992; Clough & Nutbrown, 2007; Cohen et al., 2007). The decision to conduct my research within an interpretive approach raises questions about my ontological and epistemological assumptions that I discussed earlier in relation to constructivist, empiricist and pragmatist interpretations of learning (Sparkes, 1992; Clough & Nutbrown, 2007). From a methodological perspective, the methods used in my study evolve in a unique manner because, in many ways, the participants and the research methods drive my ethnography (Denzin, 1997).

By living the experience of teacher education with my students, I am aware of the impact of education policy, school communities and classroom culture upon them. They are subject to antagonistic pulls in their teacher education (Ball, 2003) from individual encounters with pupils, to encounters in classrooms, in mathematics departments, in schools and from a national or even global level (Lave & Wenger, 1991; Wenger, 1998; Darling-Hammond, 2006, 2010). Given the complexity of this setting, there is a danger that their voices are lost within the landscapes in which their teacher education is lived. I started this study knowing that I wanted to tell my students' stories, but not knowing which students would have a story to tell. Over a two year period, critical incidents have emerged in my students' teacher education that have evolved into episodes in my data. Each episode has been explored in depth, so that the analysis of the episode is informed by classroom observation, semi-structured interview and field notes from my own university teaching. The critical incidents that occurred are frequently articulated by my students using metaphors or analogies, which I transform into themes that explain their perceptions of their beliefs about their teacher education as well as their interpretation of their behaviour in the mathematics classroom. In doing so, it is my responsibility to adhere to what Clough and Nutbrown describe as "radical listening" (2007, p. 15) so that I portray honest accounts of my students' stories. To this aim, the episodes that I write provide a narrative account of what my students have said and done within this study and within their teacher education. Each episode integrates my students' words through direct quotes of what they have said in interviews, reflective writing or emails. The aim of my narrative analysis is to provide voice to the students involved in the study, so that their particular experiences encapsulate, potentially powerfully, aspects of more general contemporary mathematics teacher education experiences (Campbell & McNamara, 2007; Williams, 2011).

At times, my research is characterised by an awkward reflexivity (Pillow, 2003) because I am walking a line between teacher educator and a researcher of my students' teacher education. Pillow claims that to be reflexive "*not only contributes to producing knowledge that aids in understanding and gaining insight into the workings of our social world but also provides insight on how this knowledge is produced*" (2003, p. 178). As in her explanation, reflexivity in this study accounts for my influence on the beliefs and behaviour of teachers in their mathematics teacher education and in doing so requires me to represent my students' perceptions of their teacher knowledge even though I am potentially influencing that knowledge as their teacher. To mitigate any possibility that I may exploit my students or fail to represent their stories honestly, I aim to research with my students and to avoid doing research to them (Pillow 2003; Clough, 2002; Clough & Nutbrown, 2007; Campbell & Groundwater-Smith, 2007; Hammersley, 2008). To this end, I aim to interrogate teacher education with them through shared experience and shared conversations.

Interviews have evolved because of incidents that my students have shared. Thus, this research is driven by my students' experience and results in willing and often enthusiastic participation from those involved in the study. I am not witnessing my students' experience, but noticing events and words that inform the study because I am sensitised to the phenomena that I am studying (Mason, 2010). Through the exploration of key episodes with my students, their own reflexivity exposes their understanding of what is happening to them and why, thus affording them the opportunity to experience the emancipatory freedom that this understanding can stimulate (Campbell & McNamara, 2007). My students are not merely describing and explaining critical incidents, but are sharing in dialogue with me as we make sense of their experiences together. This is facilitated by interviews that are stimulated by sharing one of my students' metaphors or analogies and exploring their perception of the meaning behind the metaphor and the consequences of this understanding for them in their mathematics classrooms. To this aim, sharing the construction of understanding in relation to each key episode requires all participants to understand the phenomena studied so that I can capture what my students are saying whilst allowing them to speak for themselves.

However, to interpret and construct a narrative analysis of the data that my students share with me, I acknowledge my direct influence within the data, but step back to radically listen to what they are telling me (Clough & Nutbrown, 2007). I do not pretend that I can place myself in the experiences of my students simply by recalling my earliest teaching experiences, by working alongside them or by visiting them in school; I cannot because the privilege of my experience and knowledge would expose the risk of their experiences being misinterpreted (Young, 1997). However, my narrative analysis allows me to interrogate my role in my students' teacher education and affords me self-reflexivity situated away from the contexts in which the data is gathered (Pillow, 2003). I do not attempt to remove myself from the phenomena that I am researching, but research in this way because I am directly involved in the phenomena and because this position allows me to represent my students stories to a depth that distant, non-participant action research would not (Clough & Nutbrown, 2007).

In order to allow for each participant's story to be researched in depth, this study focuses on six main participants, each of whom willingly consented to participate in the research following the emergence of incidents that subsequently informed the episodes that characterise my analysis of the study. However, my analysis of their data is informed by encounters with many people involved in the participants' teacher education, including the teachers and mentors that educate them in school, the other university tutors involved in their teacher education and the other PGCE students with whom they are educated at university. In order to protect these groups, all participants are described using pseudonyms chosen by me or the participants themselves. In being invited to participate because I have noticed an incident that is pertinent to this study, participants are asked to explore the incident with me through interviews. The incidents are special because they exemplify disturbances in my students' teacher education and so, the interviews are premised on an exploration of a theme that is potentially difficult for the student teacher and me. This method allows me to address the aim of my research by accounting for my students' experiences as they navigate the complex landscape of learning to teach mathematics (Wenger, 2013). Thus, these incidents and these participants are included *because* they exemplify disturbances and difficulties and not in spite of them. Ethically, this aspect of the study has been given careful consideration to ensure that participants are not damaged by exploring potentially disturbing incidents. For all of the participants, telling me the stories of each incident is greeted with enthusiasm, because each of the incidents exists within their teacher education and not within the study. The events that inform the episodes in this study emerge because of the students' teacher education and exploring each incident has allowed them to reflect on and justify their beliefs in relation to each incident. Although care is taken to protect each participant by ensuring that they all understand my responsibilities as a researcher and their rights as a research subject, each participant readily allows their voice to be heard (Campbell & Groundwater Smith, 2007).

In many cases, shared conversations with my students include reference to their training schools and the school-based teacher educators who allow them to share their classrooms and mentor them during the ITE year. These references provide vital clues to my students' perceptions of learning to teach mathematics. However, aside from one teacher clearly indicated in Chapter 7, the class teachers and mentors who work with my students in school have not been direct participants in this study. In order to protect the school-based teacher educators, I have not included references to contexts that would allow the teachers and individual schools to be identified.

It is my responsibility to help my students to develop their pedagogical and mathematical pedagogical knowledge (Shulman, 1987). Most of my teaching is situated away from the secondary school classroom at the university, so that aspects of teacher knowledge that relate to the context for learning are absent. Whilst this is a thread in all of the episodes analysed in this study, the nature of developing teacher knowledge also has methodological implications for this study. From the start of the PGCE course, my students learn which research informs my practice so that we have a shared language for interpreting the classrooms that they observe in school and their own early development as a teacher. In this respect, research informed and theoretically interpreted teaching is intended as a seed sown in their emerging teacher knowledge (Stenhouse, 1967). Hence, all of my students have experienced how my beliefs about mathematics education influence my behaviour as a teacher educator. This raises a concern about the validity of the data that my students' share with me, knowing my position in relation to the issues that we expose in the study. There is a danger that my students would project an image of themselves as student mathematics teachers that mirrors what they perceive as my expectations of them (Clough & Nutbrown, 2007; Brown, T., 2011; Stronach, 2010, 2011). As well as acknowledging this feature of my study, I have triangulated the data that my students shared in interviews with data from field notes, lesson observations and reflection from the many encounters that I have with them from the moment that they apply to the PGCE course. In this respect, interviews are conducted in a landscape where all participants know that I have seen them teach in the secondary classroom and that I have heard them justify their approach to planning these lessons and their explanations of the learning that took place in these classrooms. This allows me to interpret what my students have said using data that extends beyond interviews and partially mitigates the potential that their beliefs are articulated in a manner that they think I want to hear.

Wenger (1998, 2013) describes the nature of professional learning that is set in a diverse landscape. Within this landscape professional learners encounter people and events that alter their learning trajectories. Learning to teach mathematics has these characteristics because each teaching experience is set within a classroom culture, within a community of practitioners that combine with the individual teacher's beliefs about learning and teaching to create a complicated learning environment (Lave & Wenger, 1991; Wenger, 1998). Learning to teach is not a linear process of applying a succession of skills using a progression of insights. It is inherently complicated and arbitrary. Learning to teach is derived from participation in a classroom with a community of teachers and the learning is situated in the context of the classrooms that the teachers act in (Lave & Wenger, 1991, Brown, J. S., 1988). Thus my student teachers' beliefs about mathematics education are influenced by behaviours within a school or classroom. To further confound this, their beliefs about how mathematics is taught and learned is influenced by their own historical experience of learning mathematics within the culture of their own school, college or university (Wenger, 2009; Brown & McNamara, 2011). I teach mathematics education to my students at a university, situated away from the main site of learning to teach. Some of these lessons are taught in a school, alongside secondary teachers and teaching groups of pupils, but during these sessions my presence and behaviour ensures that university-based learning is embodied during those days. The situated nature of my students' learning is an inescapable feature of this study and is the reason why the data that informs the study is derived from numerous sources, gathered through my many encounters with my students. Ultimately, their stories are told through their words, deliberately captured in interviews, reflective writing and email, but the essence of my interpretation of their stories is enriched with insight from many encounters with the six students in the study and the hundreds who have come before them.

The four episodes that follow present the data and my analysis of the PGCE students' stories as they navigate their teacher education in school and at university. My research aims to ask how my students articulate their perceptions of learning to teach and how they justify their actions in the secondary classroom. I explore how they perceive mathematics teacher education from university and school teacher educators and how these influences effect their beliefs about mathematics education. The following four chapters capture my students' stories as they explain how they have navigated the landscape of learning to teach mathematics.

Chapter 4 Episode 1: Gateaux and One of Your Five-a-day

This episode tells the story of two students, Rachel and Sam, who have almost completed their initial teacher education in the traditional university-led PGCE route.

During one of my university mathematics education sessions, I invited one of my previous students, Anita, to come to talk to the current PGCE group about her experience of her first post as a Newly Qualified Teacher (NQT), known as the induction year. Having completed one year, Anita moved to a different school as Key Stage co-ordinator in the mathematics department. Promotion after one year is relatively unusual, but is not entirely surprising for Anita who has been judged to be an outstanding teacher in both of her schools. Before starting her PGCE, Anita spent four years training to be an actuary. She shared the reasons for her decision to leave the actuarial role; seeking a more fulfilling profession in teaching, albeit without the financial and corporate trappings that she left behind.

Within her current role as Key Stage Co-ordinator, Anita is a mentor to one of my PGCE students. She shared an anecdote from one of her mentoring sessions with the group. Her student, Jayne, had been spending an inordinate amount of time planning lessons in her second teaching placement with multiple layers of differentiated worksheets. The workload was proving exhausting. To support Jayne, Anita described an analogy, whereby the lesson ideas she is given by the university are very rich, requiring lengthy preparation, comparable to gateaux and Jayne cannot expect to have gateaux in every lesson, but perhaps once or twice a week she could plan a rich gateaux lesson. On the other hand, a mathematics teacher planning five lessons a day, was offering the pupils fruit; less rich, but necessary five times a day. The PGCE students appeared to enjoy the analogy, as I did. I had taught Anita to teach, wanting her to develop and work by her own beliefs about the mathematics classroom. After only two years as a qualified teacher, she was able to articulate her pedagogical stance and share her beliefs with beginning teachers.

After the session, I met three of the PGCE students, Kate, Sam and Rachel, in my office. I took some resources from our earlier sessions that were on my desk and asked them whether they were gateaux. “Yes”, unanimously, they were gateaux. They were gateaux because they “don’t have time to prepare those resources in school” (Rachel). I then wrote down an algebraic expression:

$3x(x+7)$

$= 3x^2+21x$

and asked them if the expression was gateaux. “No”, again unanimously, it was not gateaux, because they could show their pupils what to do. I asked what would happen if they put the expression on the board and asked the pupils whether the relationship was true or false. Or if they put in on the board and asked the pupils to discuss, in pairs, what the teacher had done. Or asked their pupils to convince each other that the relationship was true, in as many ways as they can. That would be gateaux because “*that’s the way my mind thinks”* (Kate) and not the way they think or the people in school, generally, seem to think. I suggested that asking questions like that was not gateaux, but was the obvious thing to do if you want your pupils to make connections between and within mathematical concepts. One student proposed that now I had suggested the questions, they were obvious, but that she would never think of that herself. I asked her if anyone in school thought like that: “*Not very often”.*

I frequently use the language of Askew’s connectionist and transmission teacher orientations (Swan, 2005, 2014) with my students as a means of making sense of a more connected model that I expose them to at university, which they suggest contrasts with the dominant transmission model seen in schools (Ofsted, 2008; 2012). The students in my office seemed to be enjoying the gateaux analogy. Had Anita opened up the possibility that they could dismiss the connectionist model, even in university sessions, as my romantic aspirational model? Was the ‘five-a-day’ diet a metaphor for transmission teaching that was necessary five times a day?

In an interview two weeks later, I reminded two of the students, Rachel and Sam about the gateaux analogy and asked them to tell me what they remembered about the talk.

> It worried me. It just… I don’t know it just makes me panic. Because I probably feel like my lessons aren’t even like the basic normal ones. I sort of saw where she was coming from, in a way, but then I didn’t agree with everything that she said, I don’t think. I suppose her idea of a gateaux lesson and my idea of a gateaux lesson would probably differ, which I kept going over and thinking she’s probably got an idea of what her gateaux lesson would be like. But… She thought its okay to have one of those a week, but my idea of a gateaux lesson is not the same as hers and I would do it more often. (Rachel)

> I kind of understood it as that perfect lesson that lesson that 10 out of 10 outstanding on everything sort of lesson, where the kids go out of the classroom, big smiles on their faces because they feel enlightened in some mathematical topic. (Sam)

> … and they've done it by themselves as well. Rather than you just telling them how to do something they've done it by themselves, like independently. (Rachel)

Both students brought the focus of their interpretation onto the pupils' learning and the reaction of the pupils, rather than on what the teacher does to ensure that a lesson is rich. From the start, both students are relaying characteristics of the connectionist classroom in their descriptions. I asked whether a gateaux lesson was about the experience of the learner. Sam's reaction was immediate:

> That's what it's all about anyway, it should be about the learning and not about the teaching. (Sam)

She elaborated further on her interpretation of Anita's gateaux analogy.

> It's nice to hear that no-one's expecting you to be perfect all the time. Especially at the start I felt under pressure that every lesson had to be brilliant. And that means you're putting a lot of pressure on yourself and everybody else is putting a lot of pressure onto you, whereas you don't actually have to be so hard on yourself…. But feedback from lessons was always quite negative, you should have done this and you should have done this and you should have done this. And you always felt that you were miles away. (Sam)

I was interested in the "*you should have done this"* phrase and the bitterness that I perceived to be behind the line. I wondered whether the "*you should have done*" was preceded by an opportunity for Sam to share her perception of her lesson.

> That comes after the lesson, the pressure is on when you're planning and delivering it. At the beginning I was probably more [concerned] about what the teacher is looking for. I think the further on I go I don't listen so much to the teacher and I take my cues from the children. I just feel like I'm more perceptive. I will tweak things slightly depending on what's going on in the classroom. (Sam)

I asked whether she would have considered a team-teaching approach with her mentor at the start of her teaching experience.

> That would have been better, it would have been nicer to have a gentler run on and be a bit more supported in the first few lessons. You could have planned them together and taught them together. But then there's a danger that you will just end up teaching like that teacher. (Sam)

Rachel did not appear to share Sam's reflective insights into the early days of the course, remembering her difficult experience of her first teaching placement almost with a shudder. She did add:

> It totally changes when you know there's someone at the back watching… even if my mentor is just marking and doesn't appear to be listening, you just change straight away. (Rachel)

Rachel described a "*learning to drive"* analogy whereby novice drivers learn to follow the code so that they can learn to please the examiner, but once they have passed, then that's when you really learn to drive. Both students openly shared examples of lessons that had been constructed to please the observing class teacher or mentor, to the extent that they were no longer their own lessons.

Rachel's experience with her second placement mentor was more positive.

> Everything my mentor says, I think I have agreed with and every time I bring up a point she says 'yes I've written that down', and [my mentor has] thought of that already. We definitely [have similar beliefs], and it's increased my confidence because I feel like I can be in the same position as her in a few years' time. The relationships she has with the pupils are the most important things. (Rachel)

She was no longer planning and teaching to please the examiner, by demonstrating the conduct that she thought the examiner was looking for. The more effective relationship with her second mentor was allowing Rachel to reflect on her teaching and identify improvements, often independently.

Rachel could not recall observing "*rich*" lessons in her teaching placements, but was not entirely sure what characterises a truly rich lesson. Although she had observed pupils having complete trust in her mentor, with the pupils completely engaged in the lesson. Sam added to this:

> No I don't think that I have seen [rich lessons], not in the sense that I kind of imagine the sort of rich tasks that you do with us that sort of build our understanding right from the very bottom… that I see fitting into a proper gateaux lesson. That's what… I wouldn't say I've seen any of that. Maybe it's because you don't need it all the time. (Sam)

Sam's perception that I have built their understanding "*right from the very bottom"* is interesting, perhaps recognising that some university sessions have focussed on building on assumptions about prior knowledge and connections within and between concepts so that pupils reason from known facts. But the expression "*you don't need it all the time"* is equally interesting. In her perception, Sam has not seen lessons where pupils construct understanding from their current knowledge, but if it was not needed all of the time, was it needed some of the time. Sam was not sure, but was aware that some lessons involve practising and consolidating ideas. In this respect, Sam was alluding to the need to develop pupils' fluency and conceptual understanding to master mathematical concepts; fluency in choosing and applying procedures appropriately and accurately when built upon a relational understanding of the structure and connections within mathematics (Skemp, 1993).

I reminded them about the conversation in my office after Anita's 'Gateaux' seminar, where we discussed questions that could be posed in relation to the expression $3x(x+7) = 3x^2+21x$. I reminded them about Kate's comment: "*that's the way my mind thinks*".

> It's just a different approach to the same problem. Eventually you will still get to that answer but it's a different way of thinking about it. Rather than just saying this is how it is… It's difficult because it is just so much easier to just tell them especially with time, if you've got to get so much done in the certain amount of time. [Time pressure comes from the] scheme of work and some classes are already so far behind. (Rachel)

Rachel's comment demonstrates her awareness of the complexity of developing relational understanding in mathematics, especially in relation to the time needed to develop a deeper understanding alongside mathematical fluency. The comment "*eventually you will still get to that answer*" suggests that Rachel understands that the mathematical equivalence of the two expressions is unquestionable, but the way that the learner's understanding of the equivalence is constructed differs from learner to learner. Time constraints feature in her illustration of why it is easier for the teacher to "*just tell them*", but not exclusively. She appears to be suggesting that different ways of thinking about this algebraic statement present difficulty, unless presented procedurally. (Skemp, 1993; Bruner 2006b)

Rachel went on to exemplify the impact of changing the learning context:

> If they are not used to that sort of approach… This year 8 class I've got they are just so used to the teacher going through something on the board and giving them a worksheet and he doesn't even mark it. Then I tried to come in and do different things. I had a recap of averages where they had to move to different questions around the room and it just turned into chaos. They liked it, but they just went crazy and said why are we moving around? They thought it was good, but went crazy…. [there was too much] stimulation and they weren't used to it, they weren't used to it at all, they couldn't cope with working [in groups] and the problem was set in context, and they couldn't do

> that because it didn’t say ‘What is the mean of these five numbers?’ like it does on all their worksheets. (Rachel)

Rachel is describing a sweeping change to the context for learning mathematics for this Year 8 class. She has experienced the pupils’ resistance to changing the classroom culture (Bruner, 1996) and, consequently, disturbing their expectations of mathematics classroom experiences.

> It was just too much…. and it has put me off that kind of thing… with that class. But I’ve spoken to [one of the tutors] about it and talked about slowly working towards that [approach] by the time I leave. Slowly bring in different ways of doing it. So that they will be ‘oh yes we’ve tried something like that before’. (Rachel)

Reflecting on the lesson with her tutor has allowed Rachel to understand how she could gradually change the context for learning mathematics. The chaotic lesson was problematic for Rachel, but it does not seem to have altered her belief in contextual or collaborative ways of constructing learning. This lesson, and the subsequent encounter with her tutor have had an impact on Rachel’s learning trajectory (Wenger, 2013), so that her practice can evolve gradually.

I suggested that Rachel’s example illustrates that I am asking a lot of them, as beginning teachers, to educate (or re-educate) themselves and the pupils that they teach if they are changing the context for learning. I asked whether it is wrong of me to ask so much. Emphatically: “*No*”. Rachel and Sam seemed certain that I was not misguided in my expectations.

> Because we're learning. And we've, even though they're not our classes we've still got to try to be the teacher that we want to be. We've still got to get something from it. (Rachel)

> And you're under similar time constraints that we've got. You've only got a year to get us… to train us to be at that level. If you had two years, maybe the first year we could spend just teaching these normal lessons in the classroom and maybe the second year we could introduce these more gateaux lessons. You've got to pile it on all at once, all the different techniques and all the different things we need to be looking at. (Sam)

> Sometimes you've got to realise that the idea of it and how it pans out in the classroom is not the same. (Rachel)

I interpreted Rachel's final comment as a plea, for me to realise or remember what the actual practice of teaching in a secondary school is like. From the start of the PGCE course I make no secret of the difficulties associated with what I am asking them to do; that to build a classroom based on experience, where connections are made in the construction of knowledge, where relational understanding builds reciprocally with mathematical fluency is altogether a more difficult undertaking than a transmission classroom that builds procedural understanding. But I also think that Rachel knows that, she knows it's hard, but wants my involvement "*because we're learning*" and because "*we've still got to try to be the teacher that we want to be*".

We talked about some of the enrichment projects that have taken place in school, which probably do not fit well into the daily practices of schools, such as an extended Art-Maths project. It is possible that they could fit, but they probably would not in the current subject model of school timetables. Perhaps those experiences are the gateaux that Anita described. But what of the 'asking not telling principle' that I teach them? Using a stimulus like $3x(x+7) = 3x^2+21x$, but asking about it, rather than telling pupils about it, is surely not gateaux.

> It is difficult… it's so much easier just to tell them…. And the pupils sometimes say 'just tell me what to do' and… 'Is it in the exam'. (Rachel)

This reveals a great deal about the context that Rachel is working in. The pupils' expectations of mathematics learning is based on their prior experience, as well as all of the other signs, symbols, codes and words that signify to pupils that *"I am learning mathematics today"*. If the pupils have experienced mathematics classrooms as places that do not require them to be resilient (Johnston-Wilder & Lee, 2010), or to endure the disturbance of 'being stuck' or confused in the act of learning mathematics (Skemp, 1993), then they will resist a more connected classroom culture (Bruner, 19996, 2006b).

Sam also offered insight into her interpretation of the transmission teacher orientation:

> I would never on something like this [$3x(x+7) = 3x^2+21x$] say this means times and do this and that and that. I would get the kids to tell me what it means and tell me what they already know about it. I wouldn't use [a transmission model] directly. Sometimes you see it in a class that are only a few weeks off their tests and you know it's almost like the only way to get a short term result is just to tell them what they need to do so that the following week they are able to reproduce it in their tests. You don't have that time to go back to the beginning and build all the layers up from the bottom. (Sam)

Rachel had observed the same phenomenon in her first placement school:

> You would have a week before the test where you go through the shadow test and they do tests that are exactly the same as the shadow test. (Rachel)

They are both describing models of teaching that they have observed, which focus on maximising examination performance, which has been witnessed in numerous lessons in England in recent years (Ofsted, 2008 2012). Neither of these student teachers respected this practice, but both were complicit in applying the same practices in their classrooms.

We returned to discussing the connectionist model in classrooms that are more likely to result in pupils gaining a relational understanding of mathematics. What was the most connected lesson taught so far?

> Probably… the *Farmer Jo* one for my assignment. There were a few who found out by the end how to find the [maximum] area of the rectangle. They had the length, the width and the area and they saw the relationship between those three numbers… yes and they could relate the [repeated rows] to the area because I've used the array… because their times tables are not very good. I'll do 8 dots 4 times for 8x4 and I think that helped as well because they were familiar with that. (Rachel)

Rachel is referring to a problem whereby the pupils are given a fixed perimeter rectangle and asked to find the rectangle with the largest area. In this context, the farmer wanted the rectangle with the most space inside for his animals. We discussed possible connections that the pupils had made and ways in which Rachel had made the task more accessible to pupils, using rods to represent the fence panels in the perimeter of the animal pen. She elaborated further on the use of the array as a model for multiplication and how it had helped them make sense of area far more than presenting the area of a rectangle as a formula.

> And that was a mistake I made on first placement that [my mentor] picked up on because it was area of a triangle and it was Year 8 set 1 so I just thought, well tell them *length times width divided by two* and it would be fine, but there were a couple that just didn't get it. Instead of drawing out the rectangle… I was just… I thought *just put the numbers in, it's not hard*. I

> wouldn't even dream of doing that now, it's just not logical. It's only because [my mentor] told me afterwards, I knew straight away as soon as he said, *oh yeah why didn't I just draw out the rectangle around it so that they could see that it was half*. (Rachel)

The observation, "*it was just set 1*" reveals that Rachel was challenging some of her assumptions about how higher-attaining learners construct their mathematical knowledge. Assuming that higher attaining pupils are readily adapting to a symbolic understanding, manipulating mathematical ideas without images or explicit reasoning has been challenged by a number of researchers (Boaler, 2009; Nunes, Bryant & Watson, 2009). However, Rachel was altering her practice based on her own experience, without any obvious knowledge of the theory (Bruner, 2006b) and research behind this issue.

Similarly, but more significantly, Rachel's experience of pupils failing to make sense of the area of a triangle, through the application of a procedure demonstrated by the teacher, has helped Rachel to reject a transmission model in favour of a more connected approach in this topic. After the lesson, she was able to realise that the formula approach was "*just not logical"* because it did not build on pupils existing knowledge of the area of a rectangle and nor did this approach allow pupils to derive "*half a rectangle"* or reason why the area would be half of a rectangle. This encounter with her mentor, "*It's only because my mentor told me afterwards"*, has altered Rachel's learning trajectory (Wenger, 2013), leaving her wondering "*why didn't I just draw out the rectangle around it"*. The potential impact of actually seeing half of a rectangle was immediately apparent to her after she had been prompted by her mentor. When Rachel said that she would not dream of teaching like that now, at what point had she reached that realisation?

> When I spoke to [my mentor] afterwards. It just made me think about it in a different way and I couldn't believe that I hadn't thought of it like that. (Rachel)

The mentor's comment in the location of Rachel's practice had immediately made sense; a simple solution to a problem that had just arisen in her classroom. This short suggestion from her mentor had clearly had an impact on her pedagogical, mathematical knowledge (Shulman, 1987; Krauss et al., 2008) because he offered an immediate solution to a problem.

This account represented, for me, a pivotal point in our discussion. Deriving the formula for the area of the triangle, trapezium and other quadrilaterals had been the focus on one of Rachel's mathematics education sessions during her PGCE induction at the university. The principle of making connections from key, known facts was being used to present a notion of area formulas that conflicted with many of the PGCE students' experience of transmission classrooms. Clearly, this experience at the university had not stayed with Rachel, she did not appear to have learned this model from her experience in my university classroom, but did she know why. The reply "*I don't know"* was uttered quietly, with Rachel almost shrinking into herself as she spoke. I took this as a cue to move on, to drop that line of enquiry. Perhaps Rachel thought that I was disappointed, which I may have been in my first year as a teacher educator, but not now, having experienced the absence of concepts from university sessions in many of the early lessons that I observe in school.

The focus of the discussion returned to Sam. Had she had a similar experience where a conversation with a teacher or a critical incident had changed the way that she thought about a concept?

> Possibly the other way around. I was teaching sequences with one class and I was teaching them to consider which times table it's based on and how the sequence moves [up and down the number line] and the teacher said to me at the end of the lesson, why didn't you just teach DNO. I was quite shocked because to me DNO is just a way to do it and they don't understand that the sequence is based on a times table and they don't understand why they do it. They don't understand why they're doing it, they just have to write numbers in that order. (Sam)

We discussed the procedural model for identifying the general term of any arithmetic sequence. The acronym DNO represents difference-number-zero and presents a staged procedure for finding the general term, which could be used and applied without necessarily understanding the reasoning behind each stage or how the general term relates to the structure of the arithmetic sequence. Clearly, Sam was surprised that the class teacher, a teacher that she had grown to respect, was suggesting a method that would lead to instrumental understanding.

> I think she thought I could have got through what I got through in that lesson a lot quicker if I'd just done DNO. She felt I could have packed a lot more into that lesson. I think it took them longer to grasp what they were actually doing and why they were actually doing it, but I like to think, in the long term, it's going to stick with them a bit more. (Sam)

Sam recognises what proponents of deeper, relational understanding have also recognised (Skemp, 1993; Swan, 2008, 2014), that mastery of mathematics requires more time initially to gain a deep, enriched understanding of concepts (Bloom, 1971), but that time is saved in the long term because topics that have been learned in a surface or instrumental manner are forgotten and need to be retaught, year on year as the curriculum progresses (Darling-Hammond, 2010). Had she seen this in her school?

> Yes. I've been frustrated if I've got a Year 7 class and a Year 8 class and they're at the same place in the scheme of work and I end up teaching essentially the exact same lesson to Year 8 as I did to year 7. And it's like, well if the teacher did this with them last year, why am I having to do it all over again? (Sam)

In some respect, Sam has already answered this question, with the relational understanding she was trying to build by relating sequences to 'adjusted multiples' she "*likes to think that in the long term it will stick with them*". Sam offered some insight into why she thought the teacher was suggesting an instrumental approach:

> I wonder if that's what it is with all pressures of, you know, keeping up on marking and writing kids' reports and having to monitor so many other things about them, then planning maybe takes a back seat and the teachers just pick the thing that is quick and easy. (Sam)

She could be right; contrasting instrumental understanding as swift, but surface knowledge, against relational understanding as slower, but deeper learning (Skemp, 1976, 1993).

The "*faster, easier*" excuse for transmission teaching reminded me of a comment that Sam had made after her first teaching practice. We were looking at the outcomes of the Increasing Student Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) report (Hodgen et al., 2009b) that indicates that secondary mathematics learning has not improved in England during the last thirty years, despite vast expenditure on raising attainment in schools. At the time Sam had asked, "*How can this be*?" I started to reply with generalisations about mathematics education in English schools, but Sam stopped me. "*No, how can it be that we are going into schools and you have taught lots of the teachers in those schools, they have experienced the university model, but they start teaching and revert to transmission; telling and teaching to the test"*. I explained, in the interview, how this comment had taken me by surprise. I asked them why they thought this might be:

> It's just easier. It sounds awful but it is just easier. (Rachel)

> I think it's… that if you don't do something for a long time, it does get pushed to the back of your mind. Like I said before, with all the pressure planning is pushed to the side. (Sam)

In some respects, the community of practice offered in mathematics education sessions at the university has most of the characteristics of transformative teacher education proposed by Zeichner (2003). Student teachers are part of a community of beginning teachers who share a development focus, that of becoming effective mathematics teachers. They meet regularly and, with my guidance, are offered a research and experience informed model of practice. Once they leave this community and start NQT roles, there are no longer sustained meetings with a shared focus for development. The early immersion in the university led culture for learning to teach mathematics declines during the PGCE course as concentrated teaching practice increases and ceases entirely once the students become NQTs. This is replaced by immersion in the culture and practice of the secondary school mathematics department (Wenger, McDermott & Snyder, 2002). Sam's reasoning is simple, but insightful, once the student becomes an NQT, they are less likely to be exposed to teacher education which models a connectionist classroom and, therefore, the model "*is pushed to the back of your mind"*. Her comment "*with all the pressure, planning is pushed to the side"* suggests that developing relational understanding of mathematics in a connected classroom requires planning, whereas the transmission model leading to instrumental understanding is quicker and easier. Rachel agrees, "*it sounds awful but it is just easier*". Once again the faster, easier excuse for transmission teaching is used. This alludes to a notion that these student teachers believe in a model for teaching that builds on pupils' knowledge and experience to construct images that lead to conceptual understanding, but they do something else. Teachers' actions in school are justified because they have to respond to the perceived pressures of time and efficiency in their daily roles.

In university sessions, I sometimes refer to the transmission model as the 'ex-ex-ex' model; the teacher provides explanation and examples to the pupils and the pupils are asked to replicate these examples in exercises. One key principle that I use to prompt student teachers to analyse this model is to ask whether a simple adaptation to the explanation stage can result in pupils making connections, rather than receiving the knowledge transmitted by the teacher. The simple adaptation is to wonder whether the teacher can ask instead of telling.

Returning to Kate's comments at the start of this episode, asking pupils about the identity $3x(x+7) = 3x^2+21x$, rather than telling them how to manipulate the expression is "*the way my mind works*". Kate claimed that she would not think of questions like that herself, although they seemed obvious once I had mentioned them. At that point in her development, Kate was trying to move away from a transmission orientation in her teaching. She was making progress by using matching activities (Swan, 2005, 2008) and true or false questions, trying to give her pupils a stimulus upon which to base their articulation of what their present understanding was, so that this understanding could be developed further through questioning and dialogue. Through using true or false statements Kate was attempting to allow pupils to understand the features of a concept by also understanding what it was not. She allowed students to discuss whether a relationship was true or false, but then reverted to telling at the point where she had planned for the pupils to share their ideas and to ask them questions that build on their responses. She could see that she had asked a good question, but had answered it herself. Developing effective questioning is a skill associated with expert teaching (Black & Wiliam, 1998; Mason et al., 2010). It is asking a lot for Kate to do this as a novice teacher. However, as Mason's Discipline of Noticing (2010) suggests, Kate may now have a 'finer discernment' for features of teaching that facilitate effective dialogue. This has become possible because she was able to reflect upon the shortcomings of transmitting knowledge, with her tutor and with her mentor, so that she has become attuned to notice her own questioning and the responses of her pupils. She is not yet an expert, with an increasing array of experiences on which to draw (Mason, 2010), but, to borrow Stenhouse's metaphor (1967), a seed has been sown in her development as a critical and reflective teacher.

I do not offer my students a model of teaching mathematics that is akin to gateaux, something rich and luxurious, or perhaps to others something frivolous, whipped and unnecessarily indulgent. What I am offering is a way of teaching that builds and utilises the pupils' powers to make sense of mathematics. This is complicated, but starts with seeds, like half a rectangle or an adjusted times table. However, Anita cannot claim that schools are offering pupils one of their five-a-day. The five-a-day metaphor implies a nutritious element of the mathematics education diet. Transmission teaching, as Hodgen (2009b) and numerous others have demonstrated, does not appear to be nourishing effective mathematics development.

As the interview drew to a close, I asked whether I should stop presenting a connected model to the PGCE students, when it often conflicts with their experiences in school.

> No. You've got to. I only understand algebra because of what we've done with you. Before I would do this do that, bang in this and get an answer. Now I know why it all works and how it fits together. (Rachel)

I recalled Rachel in the induction sessions at the university, often looking like a rabbit caught in headlights when I was asking my students to justify, reason, prove and to navigate multiple representations of concepts. Rachel had come a long way in terms of her openness to models that develop conceptual understanding and reasoning. She had also made great progress in her pedagogical mathematical knowledge (Shulman, 1987). She seemed aware that her own understanding was transforming from an instrumental to relational perception of mathematics. Her next hurdle was to remain open to the possible presence of connections within mathematics, where previously none were conceived, to allow her to continue to develop a classroom where mathematics is presented in a learnable manner. It is possible that she will resist the pressure to revert to the *"awful and easier"* model of transmission teaching once she leaves the community of university led practice. However, the question of whether this is possible or improbable depends on the nature of the community that she will be immersed in once she becomes a Newly Qualified Teacher (Lave & Wenger, 1991; Wenger, 1998).

On occasions I have glibly told PGCE students that I am not training them to be mediocre teachers, often in response to their assertions that connected classrooms are hard to achieve and usually prefixed by "*yes, but*" comments in discussions about their teaching. My colleague and I use the "*yes but*" moment to signify the point in which our students grasp that connected models for teaching are going to be very difficult to realise in some of the contexts where they are training to teach . "*Yes, but I have only got three lessons to teach this"* or "*Yes, but my class teacher said that I should assume that Year 7 know nothing about percentages"* or "*Yes, but I had to explain it again because they couldn't do it",* all echo tensions between questions that I ask them during lesson feedback sessions and the context that they are teaching in. Essentially, the "*yes*" acknowledges that they hear, possibly even believe in what I am saying and the "*but*" recognises the complexity of altering the context for learning in the novice's classroom. There are suggestions of this tension in Sam and Rachel's comments, due to perceptions of time pressures, recognition that the connected model is difficult to attain and a plea for me to understand what teaching is like in practice. Ultimately, it is Sam and Rachel who are responsible for their pupils' learning. I know that what I am modelling is difficult to achieve, but acknowledge that unless this seed is sown during their training year, ideas for a connected classroom are unable to germinate and grow (Stenhouse, 1967).

Rachel and Sam agreed that I have to continue to offer a connected model of mathematics education in my university teaching. The comment *"you've got to because we're learning"* followed by *"Sometimes you've got to realise that the idea of it and how it pans out in the classroom is not the same"*, is Rachel's plea to me to remember that a theoretical model analysed in the university is not synonymous with practice in the context where she teaches. I am aware that the university cannot be the location of praxis for these students (Freire, 1970; Grundy, 1987). I do not have classrooms of secondary school pupils learning in a culture determined by the school environment, together with the beliefs and practices of the teachers in schools. I am removed from the context of learning to teach mathematics, but I am convinced that offering them a connected model is the right thing to do, is necessary to allow them to become critical and reflective teachers, even though the resulting action that is originated from this informed reflection is sited away from the university.

Stronach's notion of professionalism (2010) positions teachers as "*uncertain beings*" pulled between aspects such as measures of performativity and notions of identity. I know that the first few weeks of Sam and Rachel's NQT year will be characterised by concerns over systems that operate in their new schools or the difficult climate for learning in Year 10's lessons and not concerns over how to create an enactive model for teaching circle theorems. But I am confident that Rachel's arrays for multiplication, or Sam's adjusted times tables will give them, at least for some aspects of the curriculum, confidence in their pedagogy. This is the seed that Rachel carries into her NQT year, so that she might be more sensitised (Mason, 2010) to aspects of pedagogy that foster a connected orientation because she is aware of her actions and her pupils reactions in her lessons about multiplication.

Stronach (2010) asks of beginning teachers, what constitutes practice that is absent from theory, with the conclusion that the distinction is responsibility. It is Sam and Rachel's responsibility to develop a skilful approach to dealing with the disturbance associated with 'not knowing' that genuine mathematical problem-solving requires. It is their responsibility to manage the students' resistance to a change in culture, if that is what they choose to do in their roles as NQTs. Equally, it is my responsibility to manage the disturbance experienced by my student teachers as they comprehend a relational understanding of mathematics built on reasoning, connections and a host of representations. Also, it is my responsibility to understand their resistance to a connected model of teaching, if this model conflicts with their beliefs as well as the culture in the schools in which they train to teach. Sam, Rachel and I share similar responsibilities. My justification for my approach to mathematics teacher education is the same as Sam's justification for her choice of a meaningful model for learning mathematics; *"I think it took them longer to grasp what they were actually doing and why they were actually doing it, but I like to think, in the long term, it's going to stick with them"* (Sam).

Chapter 5 Episode 2: Divorcing Parents and Authentic Voices

This episode is centred on the perceptions of two students, Adam and Anna, who are learning to teach mathematics in the school-led School Direct route. Like the students in the first episode, they are nearing the end of their initial teacher education and they have experienced mathematics teacher education in my university classroom alongside traditional PGCE students.

Adam and Anna are two School Direct PGCE students. Aside from opting to train in the School Direct route, they were both career-changers who carefully researched and selected their training schools. They had shared the analogy of divorcing parents to describe their perceptions of the relationship between the school-based and university-based participants in their teacher education. Within this theme, Adam talked about the authenticity of the voices of the players within his landscape of learning to teach (Wenger, 2013). In the early stages of the PGCE, he sent me an email praising the contribution of the teachers and mentors in his training school during a week of sessions known as Professional Development Learning. I was intrigued by the authenticity theme that he raised, but I wanted to hold back before exploring it further. I wanted Adam to have more experience of school-based education before probing his perceptions more deeply. This prompted me to explore the authentic voices theme in an interview towards the end of the PGCE:

> The sense was that these people who were coming to talk to us, I felt that they had great credibility. They... Part of it is pace. Because the pace in school is more frenetic than the pace at university. There’s something and it might be body language or attitude or the pace of getting going in the session, but you felt as if you were spending time with teachers who were clearly very busy, clearly very … well-developed in their profession. That seemed to

> create respect from the students. I can't remember now whether there would have been specific things that they were saying that made me think: "ah this is bang up to the minute" but you were left with the sense that they were doing it yesterday. You had that sense that this was their daily bread and butter. (Adam)

For Adam, the teachers in school are situated within the environment that he will teach in, sharing their insight into the role that they are playing on a daily basis and, hence, giving what they said great credibility to him. Pace is clearly a theme that indicates credibility to Adam, with teachers teaching students at the pace that the school environment operates; the teachers were very busy and therefore well developed in their profession. Yet, some of the academic sessions experienced at university offered a contrast to this credible pace:

> Where you were left with that sense of the uni stuff being more high-brow but a slower pace and almost a disconnected pace from the pace you've got to work at and think at in school. So… That sense of authenticity, credibility, it was almost like it was set up to… like you couldn't 've tried harder to produce that juxtaposition if you'd tried. Probably deliberately slow at uni to give everyone the chance to get into it. And some of those first sessions when we're all in that big lecture theatre where the PowerPoint wasn't working properly, it was a bit hard to get going because it didn't have momentum. It just added to that overall picture of a bit… crusty at university where it's very sharp and up to the minute in school. (Adam)

At this point, Adam is describing academic learning sessions, rather than the mathematics education sessions that I taught him, that were intended to be the main focus of the study. However, the relevance of this impression of "*crusty"* university is relevant to my interpretation of his general perception of university-based learning. The experience of academic sessions has had an impact on Adam's learning trajectory (Wenger, 2013), leaving him with an over-riding impression of an absence of authenticity in sessions at university. This contrast went far deeper than an absence of pace or "*doing it yesterday*" as Adam explains:

> I've felt all the way through that I'm in two parallel universes. There's the kind of breakneck pace of being at school and the kind of methodical, thoughtful, analytical pace of being at university and… it's hard to see where there's a connection between the two. You've got that sense that the two universes are running in parallel and we have this occasional link across from the one to the other but... the two are… they feel disconnected. (Adam)

I asked Adam whether the two universes ever joined up, whether there was a space where the two can connect.

> For me seeing you in the school context at the enrichment week was... wow… I didn't realise… it feels odd because suddenly you're in the school situation. Maybe in the early days trying to have some kind of activity that stops the two from feeling like two separate worlds… and it feels weird... because we see you out of context. (Adam)

This claim is interesting. To Adam, his university tutor being in school is "*out of context*" and it feels strange. Seeing me in school, working alongside my students with pupils in enrichment week was what: "*wow… I didn't realise*"? What had Adam not realised? That I was a teacher? That I could actually do what I was trying to teach them to do? I liked Adam's "*wow"*, it signified respect, but this left me puzzled as to why my years of secondary teaching experience prior to moving into teacher education failed to speak for itself.

The dilemma that Adam introduced rests at the crux of issues emerging in his interview. I offer my student teachers a model of mathematics education that rests upon experience (Dewey, 1938). Yet my physical separation, while teaching at university, from the site of teaching secondary school pupils, limits or even removes the opportunity for me to offer an experience based model of teacher education. Adam offers a pragmatic solution to this dilemma:

> So for example if you had come along to some of those [school-based] sessions it would have been more visible that all of this is part of some co-ordinated programme. (Adam)

For the participants of the two communities to merge geographically is Adam's simplistic solution to the issue of separation of school and university-based education. Specifically, he suggests that the co-ordination takes place in school, at the location of the teaching. Adam was quick to assert that this failed to happen:

> But you didn't and … and… Because we don't see a lot of you and we see too much of some of the other lecturers… that partnership feeling feels… I mean Anna's used the expression that it feels like a child stuck in between two divorcing parents at times. (Adam)

Anna's 'Divorcing Parents' analogy is insightful. Both the university and school-based teacher educators are invested in Adam and Anna's development as teachers, but the enactment of the involvement plays out as a failed marriage. Like the child, the student teacher is stuck with both parents, but the marriage of the university and school teacher educators has broken. To stretch the divorcing parents metaphor further, I could ask whether the parents were ever married, working as one community with shared beliefs and practices?

Adam suggests that he has glimpsed a marriage, a time when the two universes were bridged:

> What would be fabulous is seeing some of those techniques that you've shown to us- watch you do some of those in the classroom… I remember a time here where you ran through [a model for learning algebra] and said this is a great way to teach this and okay we're taking that on and trying to remember aspects of that…. To watch you do that with the kids. Because I

> got the chance to see you do a chocolate thing in the combined art maths class. And we're going back a long time now when I had two or three days observation at [a partner school]. And I saw you in the classroom with the kids doing the chocolate thing, doing the factors table. That was, for me, watching you teach the kids… so that you have credibility is an issue. I watched you do it with those kids and I saw how effective it was… and when you've done anything since I am not sceptical. I have complete trust and confidence that what you are explaining will work, will work in the classroom. I don't have the same trust and confidence when I am with [other tutors]. (Adam)

Adam makes a powerful claim, that he has complete trust and confidence in me, because he has glimpsed pupils learning in a classroom led by me. Yet, the university tutors who he perceives to be wholly situated with university-based education do not elicit the same trust and confidence. I proposed that the absence of trust may originate in the separation of subject specialisms in our secondary curriculum. Do I offer a message that he can have more confidence in because we are connected by mathematics? Adam disagreed, citing other university tutors who he has complete confidence in, "*I'll buy anything that [two tutors] are selling*". This alludes to an increasingly complicated learning landscape, where Adam is negotiating conflicts between the mathematics communities at school and at university, as well as separations between educators inside the university. Within all of this, Adam has to learn to teach mathematics so that his pupils can learn.

I suggested to Adam that the notion of university teacher educators gaining authenticity by teaching model lessons in school is flawed because the keystone of effective teaching modelled at the university is built on effective teacher-pupil relationships. I would not want my students to see a model lesson unless it was with a class of learners that I had known for a number of weeks in order to build an environment of mutual trust. I asked why my previous experience as a secondary mathematics teacher failed to give him a perception of authenticity. Adam did not answer this question directly, but returned to the parallel universes theme:

> You've still got that central question about what does the university do [to get credible tutors]. What the university wants is that we all come out feeling 100% satisfied that they will want to hang on every word that this lecturer says, they may want to disagree with some aspects of their beliefs but they will want to hear and listen and discussion will be based on a solid foundation. It won't be based on character or whatever and I feel that the university is a long way from that.
>
> One way of solving that is to take the lecturer out of the university and put them in the school situation and let the trainee teachers see, boy can they teach, then that builds the stock of those lecturers when we're back in the university situation. In the same way that school children come into the classroom with a prejudice against their new teacher and the teacher has to work on that and build the trust. (Adam)

To Adam, the solution is simple, come to school and teach so that you may be heard as an authentic voice. Adam does not acknowledge my point about building trust with a class in order to model effective teaching. Nor does this comment recognise another theme from our conversations in school visits; that of differing mathematics teacher orientations (Swan 2005, 2014) encountered in school and at university. Our conversation turned to whether the learning models studied in university mathematics sessions lacked credibility for the secondary classroom because they were encountered at the university and not in school. Yes, they lacked credibility, despite my claims that I had used the majority of the learning models that I have shared with him in my own secondary classroom. I asked Adam whether he felt that his uncertainty was shared by the mathematics teachers that he worked with in school. Again, Adam offers a rational solution:

> What you need to do is you need to have a debate… the debate where you are in the room and the mentors, teachers from school are in the room and

> one or two others are there and we say, let's be honest about the spectrum of different activities and lets just get that onto the table. The worst scenario is that everyone is very lovely to your face and when you go they say forget about that. The worry is if there's an element of that. I think people have very high regard for you, but I have high regard for [my first mentor] and I didn't like his style. Therein lies the problem, they can preface their comment with Sally's great at what she does. Then you know there's a *but* coming. No buts let's get the debate on the table. A debate about teaching philosophy between you and [the mentor] and other people, let's bring it on. We couldn't have that at the start, that *would* look like warring parents, but there must come a point in our training when we can have that debate. There's room for lots of different styles. The one thing we must guard against is the university tutor coming along and then leaves and everyone has a pop. There's some big stuff there. (Adam)

Adam's claim of "*some big stuff*" is not an exaggeration. Again, he is asking for a bridge between the two universes, whereby the participants in his learning landscape behave with integrity and honesty. Differences in conceptions of secondary school mathematics are acknowledged and debated because *"there's room for lots of different styles"* and the "*spectrum of different activities are shared"*. Adam's use of the phrase "*debate about teaching philosophy*" suggests that he knows that the issues go deeper than merely a range of activities; he is aware of orientations of transmission or connected teaching (Swan, 2005, 2014), he understands the distinctions between relational and instrumental understanding (Skemp, 1976) and is aware of the presence (or absence) of enactive, iconic and symbolic representations of the world of mathematics (Bruner, 1996). He does not say so in those terms, but I know that he understands these distinctions because he has shown me in his teaching or in his reflections upon his teaching.

In his proposal, Adam is suggesting that the current surface equilibrium in the teacher education system should be disturbed, that the symbolic order of university and school models of teacher education should be disturbed. Adam's description that this proposal is "*big stuff*" is not an overstatement. For the symbolic order to be maintained, I must, and the school mentors must, abject alternative conceptions of mathematics teaching and learning, so that the order of our belief systems can be maintained (Oliver, 2002). For me to acknowledge the validity of a transmission teacher orientation conflicts so completely with what I do and my perception of my beliefs about what I do, that I must abject the possibility that transmission teaching is an effective way of teaching mathematics. I acknowledge the presence of a transmission orientation, based on my experience and the wealth of evidence that suggests the presence of this orientation in secondary mathematics classrooms (Ofsted, 2008; 2012; Swan, 2005, 2014), but I abject the notion that the transmission teacher orientation results in effective mathematics learning. Similarly, it is possible that the school mentors described by Adam abject the notion that they can adopt a connected teacher orientation in a classroom dominated by relational understanding. It is possible that the teachers are aware that the connected model exists, but to acknowledge that this is possible to achieve in their secondary mathematics classrooms would upset the symbolic order within the school, and so remains inconceivable. It is easier for teachers to "*take a pop"* than to acknowledge an alternative notion of mathematics education to the one that operates in equilibrium in their schools. My presence in the school disturbs this equilibrium and it is Adam who navigates the fallout, knowing that "*there's a but coming*".

It was Anna, training to teach alongside Adam, who originally suggested the divorcing parents analogy. In my interview with Anna, she was keen to voice her discontent with the organisation and the structure of the School Direct PGCE. Eventually, we were able to discuss the ‘divorcing parents’ analogy in the context of mathematics teacher education.

Having made it clear that her use of the analogy had been in describing conflicting messages outside of mathematics pedagogy, I asked her whether there was a ‘divorcing parents’ metaphor in mathematics teaching too.

> Only going back to something you said today, it wasn’t until you saw me at [my second placement school] that we built up a relationship enough for us to be able to have a two way conversation and for me to be receptive to what you are saying. I think for a bit at the beginning… and even sometimes now, there’s still an element in my mind that feels ‘is she brainwashing me? She’s still trying to brainwash me. There’s some kind of cult thing going on here that I’m not party to’. I haven’t quite accepted this invitation to discovery-led learning. It’s not quite my thing. I get the idea of constructed and of connected and making it real. But it hadn’t quite connected in my mind as to what the whole discovery thing was and discovery meant less control and there was a resistance there. (Anna)

I picked up on Anna’s use of the word discovery, because I consciously steer away from the term to avoid confusion between connectionist and discoverist teacher orientations (Askew, Askew et al.,1997), preferring the term *make connections*. Anna countered this with a description of a lesson that she had observed, which she described as outstanding.

> But last week was more discovery than I had realised. I thought it was going to be word-based enquiry with connected [teaching] but actually there was a lot of discovery – it was them discovering with him just kind of connecting. (Anna)

Anna’s desire to make sense of teacher orientations and learning models in her own way had been a feature of conversations throughout the course. I have proposed a connected model of teaching, which research suggests leads to effective mathematics leaning (Askew et al., 1997: Swan, 2005, 2014). I have not offered Anna an invitation to a discovery-led learning cult, nor any cult, but I was enjoying her description. The use of the term cult suggested to me that I was offering an eccentric alternative to the mathematics classrooms that she was encountering in school, but, perhaps, more sinister than eccentric. A cult suggests a way of conceiving mathematics education that is to be treated with suspicion, that is to be othered and separated from the norm. Her perception of being brainwashed is a powerful claim; that I was somehow trying to shape her beliefs in order to control her behaviour. To have elicited such a powerful reaction, albeit a negative one, motivated me to explore this idea further. I asked her to tell me more:

> There has been some divorce, some separation sometimes between what we've done here and what you see when you go back. And I think in general there is a resistance to the way that you've brainwashed us and culted us into thinking. And we've all questioned ourselves, we've all come back and said 'why is it we're not seeing what we're talking about here in schools?' and you know… we're wiser now to know that it's because teachers tend to fall into the easy category and teaching connected is not easy. (Anna)

I asked her whether she thought that this was due to the pressures of examination performance to evaluate a school's effectiveness.

> Exactly and even though [teachers and mentors] might be a convert here… as soon as they get back into school other pressures take over and you revert back to type. And you often do see that. In [my second placement school] you saw it a lot- they didn't want to teach that way and that's where I struggled because I knew already what you would say about that lesson [that you observed] before you had even seen it. But I also had to try and go with the class teacher and the class teacher was saying afterwards that's a great lesson. Well yes, because that's the way you wanted me to teach it.
> I think they all aspire to what you are saying but it just… it feels like a pipe dream. There are other pressures that you can't possibly plan for… this is reality. (Anna)

Here Anna is exploring a similar thread to Adam. She suggests that teachers in her placement schools aspire to teaching in a more connected orientation, building relational understanding, but that the reality in school makes this a "*pipe dream*" that cannot be realised within the pressured school environment. On the one hand, Anna is citing the absence of connected teaching as "*the easy category*" and on the other she is alluding to the ad-hoc nature of schools that expose teachers to pressures that they cannot possibly plan for. She suggests that my student teachers have made a transition from questioning why they are not seeing connected teaching in schools to a position of acceptance or understanding why teachers "*tend to fall into the easy category*". So where did this leave Anna, caught between the two opposing parents?

> That's the turmoil that I felt I was in. There's still an element of me that wants to teach like that [in the connected way]… an element that sees that's where I should be trying to get to… I'm a teacher who's still doing the doing instead of letting my pupils do the learning. (Anna)

Anna's perception of her position at the end of the course is consistent with my own observation of her teaching. Attempts to create a more connectionist orientation were seen in short episodes of asking, rather than telling, but these attempts were short lived because Anna reverted to explaining and, in essence, answering her own question. Like Adam, it is Anna who has to navigate this tumultuous landscape. Despite her scepticism that I am trying to inculcate her into a connected teacher orientation, she is open to the possibility that she can try to get from a transmission teacher-led orientation towards a connectionist orientation that is centred around pupils making connections, "*letting my pupils do the learning".* Anna appears to believe in a more democratic model (Dewey, 1938), whereby she and her pupils contribute to the learning experience, but maintains control by "*still doing the doing*".

Later, Adam talked about his perception of the divorcing parents metaphor:

> You're watching this being played out. I'm paying nine grand for this professional training experience and I'm watching it being worked out… It's awkward because you want that, you don't want it to be sterile, you want a bit of grit in the system. You pay nine grand and what am I getting for my nine grand? (Adam)

Adam was the only student that I interviewed who questioned whether he was getting value for money in his teacher education. Policy reforms have provided Adam with a choice, opened up the ITE market so that the product that he chose must give him value for his money (Ball, 2013). He wants a *"bit of grit in the system"* but at what cost? Earlier in the interview, he used phrases that suggested that he felt that the university was responsible for the quality of his teacher education experience, "*what the university wants is", "what does the university do*" and so on. Adam continues to question the effort of the university as he describes his negative learning experience in the first phases of the course:

> When you start your teacher training you are told so much about the importance of scaffolding and how critical that is to making sure the children are within their… zone. Yet I felt zoneless for a lot of that time and I felt miles away from any comfortable zone in that October November December time. So you think, where was the effort that the university have put in to scaffold my experience. So that I was in a protected zone that would gradually expand out as I became more… and I remember my mentor saying 'I'm just chucking you in at the deep end here marking books, parents evening, you're getting the lot, because you'll have to get the lot and you're going to have to cope with it'. If someone had said to me, we're going to expose you to all these different things in the first week and what I want you to do is tell me where you feel comfortable and where you don't. And what we'll do, we'll make sure that we extend your feeling of uncomfortableness, but we'll do so in a way that has your agreement and we'll do it with you rather than at you. And I didn't feel that that was a collaborative exercise. I felt that it was done to me. I received lots of stuff and I've been the one who has had to modify myself to the point where I've started to accommodate…. as long as my sanity stays intact, which was looking dodgy around Christmas. But I've come out the other side of that and I'll be stronger for it. But there's also a deficit, it also takes away a piece of resilience. (Adam)

He asks where was the effort that the university have put in to "*scaffold my experience*", whilst at the same time describing the mentor's decision to chuck him in at the deep end. Adam uses "*the university*" to position the blame for his negative experience in the first term, yet describes the cause of his difficulties in terms of what the school-based educators did "*to him*". His analogy of the two parallel universes that feel disconnected remain so in his conception of professional learning and pedagogy, but seem to have overlapped in receiving blame for his perceived difficulties in his training. The school-based mentor chose not to follow the programme suggested by the university, but the university is questioned, "*where was the effort that the university have put in to scaffold my experience".* He chose an ITE course led by his school, but seems to locate the responsibility for the quality of this product with the university. Adam expanded further:

> My mentor was absolutely right that in some of the things that he said, that you have to go through that experience and come out the other side or you won't know what your boundaries are. When you've got a 10% dropout rate in the first year and a 40% dropout rate by the end of the fourth year, something's not right…what's astonished me is that there isn't more effort … from the machinery of teacher training to try to understand why so many people are disaffected with it. (Adam)

The "*effort of the university*" shifts, in this comment, to the "*effort from the machinery of teacher training*". This may suggest that Adam is aware of the wider involvement of the university, school, government and government agencies in teacher education in England. Although the interview had strayed away from mathematics education and differences in pedagogy, I was interested in Adam's broader perception of his teacher education. He had "*lost a piece of resilience*" in the negative experiences of teaching in his first term, yet was clearly resentful towards the university and not the school who had made no attempt to scaffold his experience. This left me wondering whether his perception of mathematics pedagogical influences was clouded in the same manner.

It was possible that my interpretation of Adam's blurred focus of blame positioned me within the collective "*university*", rather than the person leading the university mathematics teacher education that I set out to consider in this study. I asked Adam whether there were any examples of the divorcing parents getting along well.

> I think there's a danger in thinking that you're the only connecting point between the two worlds. There was a lot of bitterness from [another university tutor] towards school … [The tutor] was almost pointedly saying don't ask me ask them. The divorcing parents… But the bitterness was from [the tutor] and not you… We've seen it from the other side too, from school. Some people diplomatically smooth, some people make no secret. (Adam)

Adam's initial comment suggests that I am not wholly situated in the collective university world, but that I am somehow the connecting point between the two worlds. We discussed some of the immersion days in schools that I have designed with mentors and mathematics teachers from school. Was this an example of when the parents got along and shared practice or was I simply the accompanying parent?

> The [day in school in the induction period] … fire of doom… number flowers. I didn't gain as much] as I did in the [mid-programme school] day, the questioning day. It's all in the timing. I think we all knew so much more by then. The first one probably was useful, just the act of rehearsing what you might do in the classroom situation. Because you've got to start somewhere, but the second one sticks in my mind as being so much more useful. The later ones have all come across as more coherent. They've run smoothly, they've purred along ... I thought the programme could and should have had more of that. (Adam)

On these days, the two worlds intersect. The structure of the programme is planned with the teachers from school and the pedagogical focus is worked out in collaboration. I am a visitor to the school, along with my students, but for those days at least, I have some legitimate participation in the school mathematics community (Lave & Wenger, 1991; Wenger, McDermott & Snyder, 2002). My presence in the school is not weird or disturbing to students like Adam. We were all working together and hence, it seems, the days "*purred*" along coherently. My earlier incarnations of these immersion days were less collaborative. I needed some pupils for my students to work with and so I needed to borrow them from one of my partnership schools. The structure of the programme was organised by the school and I made no attempts to include teachers from school in the design of the pedagogical focus for the day. The school provided the context and I provided the mathematics pedagogy to be learned. The more recent, collaborative model has evolved as my relationship with partnership schools has developed; mutual trust has allowed us to collaborate, to offer alternative or complementary perspectives on mathematics learning respectfully. Or at least that is how it feels on the day.

However, each one day immersion is a far cry from the debate suggested by Adam earlier, in his plea to expose the apparent differences between university and school perspectives on teacher education and risk upsetting the symbolic order within the system. The divorcing parents remain polite on the occasions when I represent a connecting point between the two worlds. Meanwhile, Adam and Anna are left to navigate this complex learning landscape.

Chapter 6 Episode 3: The Caterpillar Method

Both of the previous episodes tell the story of beginning teachers moving towards the end of their ITE courses. This episode turns to two Newly Qualified Teachers, Ben and Luke, who I taught in the traditional PGCE course. At the time that they completed their ITE course, the School Direct route was not available to them.

Ben and Luke recently completed a year as Newly Qualified Teachers in two separate, mixed comprehensive schools. Previously, in the second half of their PGCE, they shared a placement in a partnership school, sharing a mentor and team-teaching some of their classes. In my final tutor observation, towards the end of the PGCE, I observed them teaching, Ben with a higher attaining Year 7 class and Luke with a middle attaining Year 8 class. In the discussions that followed the observations, we discussed a method for solving simple linear equations that they called 'The Caterpillar Method', which is similar to a 'function machine' or flow diagram (Booth, 1984 cited in Nunes, Bryant & Watson, 2009) approach to solving simple equations, but with the stages within the 'function machine' represented in the sections of the caterpillar. The method had been modelled by an Advanced Skills Teacher (AST) in the placement school to demonstrate its usefulness in encouraging pupils to identify the correct order of operations on the 'unknown term'. The method then encouraged pupils to find inverse operations by travelling the opposite way along the caterpillar and using the inverse operation in the reverse order. Earlier in the PGCE course, I had introduced Ben and Luke to a method for making sense of expressions and linear equations where the variable or unknown expression had a discrete, concrete representation such as the 'number of sweets in a packet' or the 'number of cubes in a bag'

(Prestage & Perks, 2005; Mason, Graham & Johnston-Wilder, 2005; Duke & Graham, 2007). This model was chosen to demonstrate the enactive stage of learning (Bruner, 1996, 2006a) to use and manipulate expressions, ahead of the more typical symbolic stage of manipulation observed in many secondary classrooms (Ofsted, 2008; 2012).

At the time, Luke was going to "*go with the caterpillar method"*, why not, if it worked for the AST it was "*good enough for him"*. However, Ben was finding his own way of prompting the pupils to learn how to interpret and solve linear equations. Before his lesson he had said "*I believe in the cubes in a box model, but I'm doing this*", indicating the caterpillar method. But in the lesson observed, the learning was built around neither method. Pupils had become experienced in constructing expressions from descriptions such as 'I think of a number, add eight and then multiply by 5' to make expressions equivalent to 5(n+8), albeit in forms that may not follow formal algebraic conventions such as (n+8)x5. Ben's aim was for the pupils to be able to 'read' expressions and to be able to interpret the meaning of the symbols used. The pupils that I talked to were confident that the value of the expression would change according to the value of the number that the teacher had thought of, thus demonstrating knowledge of the symbol as a variable.

During the lesson the pupils were encouraged to make the transition from the variable n, to n representing a specific unknown number, because Ben told them the value of the expression. For example, 'I think of a number, add eight and then multiply by 5' is followed with 'and I get 60' so that a specific value of n is required, in this case n is 4, because it is the only possible input that will give the output 60. For the pupils that I observed, accurate construction of the equation was unproblematic. Interestingly, when solving the equation the pupils were using a combination of inspection, working the value out mentally, or by a combination of inspection and solving using a formal and informal method. In the example above, one pupil wrote (n+8)x5=60, correctly interpreting the meaning of the 'I think of a number' statement and then wrote, underneath the equation, n+8=12, followed by a final row of n=4. When I asked the pupil to explain what he had done, he said "*because twelve times five is 60 and four add eight is twelve*". I found the correct interpretation of the structure of the equation striking, especially when compared to pupils that I had observed in other lessons, who could construct a function machine, once instructed to do so by the teacher (Ryan & Williams, 2010), but could not interpret their answer in relation to the original equation or in terms of finding a specific unknown value. Further conversations with pupils revealed their fluency in constructing and interpreting the equations from the 'I think of a number' problems that Ben had provided.

I interviewed Ben and Luke, separately, at the end of their NQT year. With Ben, I began by reminding him of our "*I believe in that, but I am doing this*" conversation at the end of the PGCE, which he remembered clearly. I asked him how he had approached this part of the curriculum in his NQT year.

> Probably a mixture of both. I still do it now… I think of a number…. We spent about 3 or 4 lessons on it this term [in Year 7]. It's just… I think that if they can read an equation then they can solve an equation and they can understand what it means. Looking at 4x+3=15 doesn't mean a lot to anyone, well perhaps to us, but it's to be able to think oh 'I think of a number, times by four and add three to get 15', then it makes a lot of sense to them. (Ben)

Ben’s theme of finding models that “*makes a lot of sense to them*” is consistent with his explanations at the end of the PGCE course. He explained a similar approach used with his Year 8 pupils:

> Very similar way. And I did it with my year eights as a recap. The good thing this year is I have had two similar Year 8 classes so I have been able to try a technique, then if it’s worked I can try it again and if it hasn’t worked I’ve been able to tweak it, as long as I swap around the classes. Not always the same class who are the guinea pigs. That’s helped a lot. (Ben)

Some of the lower attaining Year 8 pupils had struggled, he said, due to weaknesses in numeracy:

> I kind of fell back onto function machines… erm…. Just to get them doing operations on numbers and then to get back we do the reverse. (Ben)

I asked whether he was exposing his pupils to different representations:

> I did, I did paperclips in a pot. That was with year 8s. Just when I introduced expressions, coming up with expressions. I have a number in a pot and I chuck more in… yeah, and hopefully getting them to pick their own, but a lot of them liked ‘I think of a number’. But a lot of them were coming in and saying, can we play I think of a number? (Ben)

Ben is describing a scaffolded approach here (Bruner, 1996). Pupils with numeracy weakness, who cannot sense that n+8 must be 12 in (n+8)x5=60, were offered a way of working with the inverse relationship between addition and subtraction or multiplication and division using function machines. Ben believed that they were not ready to interpret the multiple connections within the structure and manipulation of (n+8)x5=60 because of their lack of number sense and their inability to recognise inverse relationships. Hence, the scaffold, the stages incorporated in the interpretation of the equation were separated out because, he felt, the pupils could not incorporate inverse relationships into their understanding of the equation until they had an intuitive grasp of the reciprocal relationships between addition or subtraction or multiplication and division. Meanwhile, most pupils were offered two approaches to constructing expressions; a physical model using paperclips in a pot, where the variable or unknown term was the number of paperclips, together with the 'I think of number' approach that was very likely to be familiar from their primary education. In both cases, Ben was offering the pupils a way that would hopefully "*make sense to them"*.

In the lessons that I have observed Ben has been sensitive to the pupils' powers in making sense of the mathematics in front of them. In their lesson described at the start of this episode, his pupils were merging formal and informal methods for solving equations. So the scaffold used by Ben is probably devised in response to the difficulties that he perceived his pupils to be encountering. He has assumed that solving (n+8)x5=60 is "*too much maths"* to deal with at once. However, the scaffold offered is his decision, he decided that focussing on the effect of multiplying by five and dividing by five is what the pupils need to prepare themselves for mastery of (n+8)x5=60. In this respect, Ben is separating the procedures with the aim of practising them so that they can be eventually combined into one overall method for solving the equation. I am not sure that this practice complements the overall approach described by Ben. Did these pupils need a staged procedure, or are they looking for a way of making sense of the problems enactively or iconically? That is, do they need an experience that relates to 'five times bigger and five times smaller gets the number back to where I started' or images that allow the students to picture the concept? Ben is on his way towards becoming an expert, and rapidly, because of his reflective approach, collaboration with colleagues and relentless focus on his learners. However, the application of this scaffold may be flawed, in that when the scaffold is removed (Bruner, 2006b), what remains are a series of procedures rather than an idea (Dewey, 1938).

Luke's interview as an NQT compounded the themes that I had discussed with Ben. I had observed his year 7 class prior to our interview. He was using the 'cubes in a box' approach to constructing expressions by introducing the pupils to a sealed box of pens, where c represented 'the number of pens in the box', a pupil had suggested using c to represent the number of pens in the box. Later, Luke explained that he had spent some time working with the pupils on the nature of the symbol and they seemed to readily adopt the phrase 'the number of…', attempting to avoid the letter-label confusion often observed in secondary classrooms (Hart, 1981: Hodgen et al., 2009a). He then introduced two identical boxes of pens and asking them to write an expression for the total number of pens. He wrote the pupils' responses on the board:

c+c   c+2   2c   cxc

He then explained that there were actually 10 pens in each of the boxes. At this point I had expected Luke to allow pupils to discuss which expression was correct. Having gone to the trouble of exposing the misconception and making this explicit with "*they can't all be right"*, I had expected a period of debate, where the pupils were encouraged or prompted to resolve that only 2c or c+c would result in the correct number of pens (Swan, 2005; Hodgen et al., 2009b). However, at this point Luke demonstrated that only c+c and 2c had the value of 20 and quickly erased the incorrect expressions c+2 and cxc. I asked Luke about this afterwards:

> When I was doing it, I could have elaborated on this a little bit more. What I've done in the past is brought up a misconception, tried to explain it and confused them even more. So what I thought I would do is prove to them that's right, so 2c and c+c is right by using substitution. Then eliminate the other ones. I didn't want to really spend… I could, if I had wanted to, delved a little bit more into why they were wrong. I could have proved that it is wrong, so for example if c=10 that would have been 12 [c+2] and then obviously cxc would have been…. And maybe someone would have got that that's c squared. I really could have done that, but I didn't want to lose them when I thought I just had them… you know if you listen to me for more than 5 minutes, then you just switch off. I wanted to rush that along a little bit. (Luke)

In this case, Luke seems to be leaning towards a connectionist classroom, but has reverted to correcting, rather than addressing the misconceptions, during the lesson for fear of talking too much or confusing his pupils. In this respect, Luke is recognising some features of the connectionist model by explicitly exposing a conflict, but has not handed over responsibility for resolving the misconception to the pupils. His use of "*I could…", "I could if I had wanted to…*" and "*I really could have done*…" reveals that Luke's interpretation of his role is that of the 'fixer' or 'prover', which is characteristic of the transmission orientation (Askew, 1997, 2002). Time pressure is clearly a feature of Luke's analysis of the lesson, as seen in his final remark.

We discussed this part of the lesson further. Was it possible that the pupils could have resolved the misconception themselves?

> I think if I actually posed the problem and I said ok then we've got 2c we've got cxc, c+c and c+2. I'm going to tell you the value of c, you go away and

> work them all out for me, then 'which ones right'. I don't know if that could have been a better way of saying it. (Luke)

Again, Luke describes what *he* could have done in posing the problem and then struggles with the notion of letting them go, "*you go away and work them all out"*, but, perhaps significantly, he adds "*for me"*. Still further, this description is about the teacher's performance, is this "*a better way of saying it*", which contrasts with Ben's focus on how the pupils make sense of the mathematics. It is possible that Luke's interpretation of pupils' learning relies on how he presents the learning to them. Whereas Ben's desire for the mathematics to "*make sense to them*" contrasts with Luke's focus on his role as a teacher. We talked about the strength of creating a conflict between correct and incorrect expressions and I asked Luke whether it might be possible for the pupils to resolve the conflict. He agreed, "*yes, it is*" but did not elaborate on his answer. I asked him to tell me more about the boxes:

> The boxes. I read an article when I was at university called 'Ban the Equals Sign' and I wrote part of my essay on it and it really brought home to me that they do have lots of misconceptions and they come into the classroom and they write down 2c+3=9 and they just take away 3 and divide by 2 and, for me, there's no real understanding of what they were doing. For [this class] I haven't shown them an equals sign yet and I'm still trying to bang away on what it means. And then hopefully, I'll be able to work backwards. I did it with year 10 a low ability class, the bottom 20% of kids, and their retention might be really poor, however I spent so much time on the expression side of it and on substitution that when I did bring in the equals sign it was a much more natural progression. So that's why I did the boxes, because it fits in there. (Luke)

I wondered about how the boxes "*fits in there*". I asked Luke whether one box of pens with two extra pens might help pupils make sense of the expression c+2. He thought about this proposition and the said "*Yes. That would have been better"* (Luke), and although he did not elaborate further, his agreement seemed to be sincere. Introducing a learning model, and then removing it before the pupils have had chance to make sense of the mathematical structure within the model, had been a subject of discussion during Luke's PGCE course. Luke had made an attempt to provide pupils with a way of learning about variables and unknown quantities enactively, using a physical representation that he had grown to value through his own classroom enquiries. However, the transition towards a symbolic model, using the formal symbolism of c+c or 2c was made hastily, without an attempt to allow the pupils to experience the iconic stage of learning (Bruner, 1966). Luke's reference to "*Ban the Equals Sign*" (Prestage & Perks, 2005) suggests that he is aware of pupils' misuse of the equals sign in seeking an expression that equals a single answer, as it might in working with numbers, rather than representing a notion of equivalence as the equals sign does in algebra (Hart, 1981; Hodgen et al., 2009b).

Luke alluded to time constraints in the phrases "*rush that along a bit*" and "*didn't really want to spend*". However, his earlier comment that he "*didn't want to lose them*" when he thought that he "*just had them*" suggests that he has experienced the complexity of fostering relational understanding through his teaching (Skemp, 1976). In this respect, moving on quickly is akin to papering over the cracks exposed in the responses c+2 and cxc. The correct expressions have been validated, but the misconceptions that led to the incorrect responses are unlikely to have been resolved by the teacher's demonstration that substituting 10 works for 2c and c+c (Hart, 1981). Yet, delving more deeply into the thinking behind the responses cxc and c+2 and teaching in a manner that prompts pupils to resolve these misconceptions is a process that Luke is avoiding; perhaps because he recognises that a deeper understanding requires time in order to master these concepts or perhaps because he is aware that some of the pupils have learned the meaning of the expression already, and

Luke does not "*want to lose them"*. Lose them from what? Is it possible that Luke is aware that the pupils' interpretations of the meaning behind the equivalence of 2c and c+c is subjective, dependent upon the way that the pupil makes sense of secondary mathematics, or even the non-equivalence of c+2 and 2c? If it is, then Luke's reluctance to "*lose them"* suggests that he does not want to expose the messiness of this subjectivity, when it is quicker and easier to rely on the objective mathematical truth that 2c is identical to c+c. Luke's approach is touching upon the principles of the connectionist orientation (Swan, 2005) by exposing pupils' misconceptions, but, in this lesson at least, the resolution of this disturbance is dependent on the teachers' explanation.

In his comment "*there's no real understanding of what they were doing"*, Luke reveals that he wants his pupils to gain a relational understanding of the mathematics that they learn, but phrases that focus on him "*proving"* or him "*showing"*, suggest that he has maintained control over how the structure within the concepts is represented, which is more typical of a transmission teacher orientation that stimulates instrumental understanding (Skemp, 1993). This conflict was apparent in other aspects of our discussion. Pupils in Luke's lesson had used handheld devices to enter responses to questions that Luke had set that required them to evaluate algebraic expressions by substituting the variable for a specified number. Using the electronic device, pupils knew immediately whether an answer was correct. We talked about two pupils that I had observed eventually entering all of their answers correctly, but writing nothing in their books. Luke wrestled with the absence of notes:

> Yeah this is my problem. I think- am I bothered [about what they are writing down when] the way they're thinking mathematically is wonderful.
> However… I'm in a bit of a conflict- are they ever going to look at their book

> again and should there be some solid notes there with some solid examples. I would love to do an experiment where they don't write anything down they do it all through whiteboards or through systems and see how their results are at the end of the year. I don't know… I don't know. (Luke)

I was particularly interested in Luke's use of the word conflict. We talked about the discussions that I had observed between the two pupils, and the level of reasoning used to rectify incorrect responses. The pupils had indeed demonstrated wonderful mathematical thinking in making sense of the expressions, rectifying errors through dialogue, which was, at times, good humoured argument so that they could eventually evaluate them accurately. Yet the absence of evidence that this learning had taken place was troubling Luke:

> I can prove that they've made progress. However, if I asked them to do it again next week, can they look back in their book… this is another Claxton word… I want them to be more resourceful so that they're not asking me. I do want them to have it in their books. I do still want the model to be there. (Luke)

Towards the end of this extract Luke has resolved his conflict, at least for the time being. He is attempting to position the notes in their books as a record of learning that can be referenced by the pupils. Though his use of '*I can prove that they've made progress'* alludes to the issue that he faces in school, which is to provide evidence of his pupils' learning for people other than the teacher and the learner. Current practice of school leaders and inspectors uses book scrutiny to assess the effectiveness of learning that takes place in Luke's classroom (Ofsted, 2012).

Luke is wrestling with the issue of ensuring that there is evidence of 'learning' in the pupils' books, which is starting to conflict with his evolving teaching practice.

> When I first started this year it was a case of I want to be…. and this is going to sound really egotistical now… I want to be at the front. I want them to take what I'm doing, listen to it and replicate it in another problem and set it out the way I've set it out. And yes, there's still elements of it at times when I do want them to set it out a particular way. Like in algebra because it will help them in the future. But I've taken a step back from it now and hopefully letting them be a bit more creative. I'm trying to instil in them that it's okay to make a mistake…. But eventually I want them to get it right. (Luke)

As well as clarifying his desire to move away from a transmission teacher orientation, Luke recognises the need to "*let them be a bit more creative"* in a climate that values the individual way that pupils present their mathematics, one where mistakes are valued as part of the process of learning mathematics. Luke is describing the iconic stage of learning, whereby pupils make sense of a concept informally, using their own notes, signs or 'icons'. The aim that "*eventually I want them to get it right"* suggests that he perceives the ultimate goal of symbolic understanding, using and reasoning formal mathematical signs and symbols, but that the transition towards symbolic understanding is preceded by experiences that are less succinct and elegant than formal, symbolic mathematical representations. Luke's comments are characterised by conflict, teased by the allure of a classroom built on relational understanding, but caving under the weight of time and complexity to resort to instrumental processes. He is attracted to the principles behind the connectionist teacher orientation, but is torn between his perceived expectation of formal, conventional mathematics in pupils' books and less conventional, creative problem-solving that develops pupils' depth of understanding. And why would he not be struggling with these conflicts? He is a newly qualified teacher, developing his pedagogical practice in an arena that could nurture uncertainty in beginning teachers' beliefs about pedagogy (Stronach, 2010; 2011).

We talked about Ben's perceptions of support from teachers in his mathematics department. I asked him what his colleagues thought about his 'pens in a pot' approach and his approach to avoiding the equals sign in the early stages of algebraic manipulation. He had no idea, but then added:

> We're huge on sharing practice, we share all sorts, we're always sending stuff out by email, but I still feel a bit like a snotty little NQT and I don't know if I can go and say "*this is awesome try this"*. (Luke)

This suggests that Luke's perception of sharing practice is sharing resources by email rather than developing pedagogical approaches together. He is not aware of the approach taken by his colleagues in developing pupils' early understanding of algebraic generalisations, but this does not appear to have affected his resolve to use the 'pens in a box' and 'ban the equals sign' approach:

> Unless someone comes up with a much better way, I'll never teach algebra, especially substitution, in any other way… because I've had so many decent results from it. I'm dead impressed. (Luke)

And what about other areas of the curriculum?

> I'm a big believer in giving them a model first. If I can get outside and show them something, I will. The pie chart thing that [one of the university tutors] showed us… It was top set year 9 and I didn't want to spend too much time on it because they'll get it like that, but I wanted to give them something that they can think back to down the line… its worked. (Luke)

Again, Luke perceives significance in making sense of concepts through physical experience. Here, he is describing a pie chart constructed from people standing side by side in a circle, so that his pupils were able to experience the need to have equal arc lengths for each pupil and that each of the thirty pupils had 'twelve degrees per person' when constructing sectors. Luke was clear on the goal of the physical activity, albeit without spending too much time on it because he expects higher attaining pupils to "*get it"* swiftly. Having "*something they can think back to*" later suggests that Luke wants his pupils to have a physical model that they can imagine when making sense of pie charts in the more conventional classroom environment. As before, this comment merges conceptions of both connectionist and transmission teacher orientations; an enactive representation is constructed collaboratively so that pupils are able to imagine ideas symbolically, but time cannot be wasted on this type of activity for higher attaining pupils, who he expects to make the transition to imagining swiftly. It appears that other colleagues are aware that Luke is using enactive approaches, because they can see him taking his pupils outside to construct pie charts or compiling boxes of pens for his pupils to use, but the aim of these activities is not discussed.

Ben's mathematics department is smaller than Luke's allowing him to discuss teaching and learning readily with colleagues so that, "*We all work together and it's a really small school. It's a very close-knit community*". He describes regularly popping in to each other's classrooms because '*everyone is really helpful'*. This echoes Ben's remarks when he was applying for his NQT role, actively seeking what he perceived to be a community school. Ben is able to articulate his beliefs about his developing role succinctly "*It's all about the learners, that's what it's all about isn't it?*". We talked about his openness to approaches that helped to foster a classroom built on reasoning and conceptual development from the start of the PGCE. I asked him why he thought he absorbed notions of a connected classroom without reticence.

> I was taught maths in this room, and it was very, you know, these are the formulas get on with them and fortunately I just picked them up understood what it was. But there were still people in my class who just couldn't… why have you put those in why did you do that? It just makes sense, build it up from basic principles. Why anyone would do any different? I don't know… Fortunately it didn't make that much difference to me, but I would have enjoyed seeing how those formulas were derived. So I would like my students to experience what I didn't. If that makes sense. (Ben)

Ben has a vivid recollection of the transmission classroom that he experienced and how this approach was ineffective for many of his peers. I was interested in the phrase "*I just picked them up, understood what it was"*. It is possible that Ben developed conceptual understanding of the procedures that were transmitted in his lessons independently. In fact, a number of other PGCE students said that they aspired to Ben's level of insight and mathematical thinking, his ability to see connections where they saw procedures.

We discussed a lesson that I had seen Ben teach towards the end of his NQT year, which exemplified his attempt to take a concept and "*build it up from basic principles"*. Pupils were reasoning approaches to finding the area of a triangle, by physically cutting rectangles in half. Ben had established a clear understanding of the area of a rectangle articulated as 'n rows of m squares' as pupils made the transition to the formula 'area equals length times width'. Reasoning the rectangle area formula was the basic principle that Ben wanted his pupils to use to derive understanding of how to find the area of the triangle. This is an approach that Ben had used from his first PGCE teaching placement, which he subsequently adapted for the Year 7 class that I saw.

> I've moved on to get them to draw squares in their books and cut it in half and find the squares in the triangle and they get that it's half the area of the square. And I challenge them to say does that work for the rectangle, any

> sized rectangle and you get the odd few who say no, so I challenge them to find me one that doesn't work. I like a visual representation. The kids in the class seem to respond well to being able to see something. The rectangles, that's the foundation really and building on from that…. The kids responded to it as well. (Ben)

I had encouraged Ben to reflect on the approach of reasoning from key facts in making sense of the area of a triangle, parallelogram and trapezium during the first few weeks of his PGCE course, almost two years before this interview. I wondered why he was so open to this approach, whereas other students were reluctant to reason from the area of the rectangle, waiting until they had tried and failed by telling the pupils a formula, or in some cases, never building pupils' knowledge from key facts.

> Hindsight's a wonderful thing… I don't know how you would get [other PGCE students] to do it from the off, because as soon as I saw it I said 'that's it, that's what everyone should do'.
> You've just got to be aware… I would hate…personally the reason why I've absorbed everything and took it all on board is that I would hate to be in that position, where you haven't prepared and it really goes down the drain. (Ben)

Clearly, Ben is motivated to develop his insight into connected approaches to learning mathematics because he does not want to be in the position of learning from his pupils' failure to learn, wanting his pupils to experience learning mathematics in a manner that he did not. Ben reflects on teaching models ahead of his teaching.

Luke, on the other hand, articulated the influence of university sessions differently, when I asked whether the connected model used at university went with him into his NQT classroom:

> At the time I would've said no. Now, yes, without a doubt. Without a doubt…. I don't know why… maybe I didn't appreciate the importance of what I'm trying to do until I had made the mistake myself. And then I went back and

> thought about why they didn't get that, if I've just glossed over something. You do it in your NQT year. I would like to think that I haven't had many, but I've had lessons when I've just gone, this is how you do it off you go and it's been disastrous. If you tell them a formula it just goes in one ear and out the other. They might be able to replicate it in that lesson but ask them to do something a week later and they can't do it.
> I think it's because I needed to learn. I needed to make the mistake myself and then go back to it. (Luke)

Here, Luke demonstrates a clear understanding of the limitations of transmission teaching and the surface understanding characteristic of procedural knowledge. At times, he has used a transmission approach, but clearly wants to reduce the frequency of the lessons characterised by telling the pupils what to do and monitoring them as they replicate what the teacher has demonstrated. He suggests that he needed to learn from the experience of transmission teaching being ineffective, before reflecting on more effective approaches. This led us back to how he had wrestled with the two models for solving linear equations, building on from cubes in a box or the caterpillar approach.

> When I read that article, which I never thought I would say because the literature was not my be all and end all. However, when I read that article it resonated quite well with what I had done in the past. And I had seen things in school as well. In school I had seen an AST who had been Ofsted inspected and they said it was fantastic when they had shown Mr Caterpillar and I had tried Mr Caterpillar and I didn't like it and I tried it with my bottom set and they didn't like it… so I'm going to go back to a more traditional way to solving equations. The caterpillar… it stops [with unknowns on both sides of the equation]. First they have to rearrange and then use the caterpillar. When I saw the AST do it, I thought okay so that's the way I'm supposed to do it, if that's what actually works. But then when it didn't work for me, I thought okay I will go back to what I think is right. (Luke)

There are a number of revealing phrases in this section. Firstly, as a PGCE student who resisted all but the minimum engagement with mathematics education literature, Luke had valued the 'Ban the Equals Sign' (Prestage & Perks, 2005) article because the authors' message resonated with his difficult experience of teaching algebra in his first teaching practice. Secondly, he accepted the validity of the "caterpillar" method because Ofsted had praised the use of the method in the AST's lesson. This validation would provide a very convincing justification for the caterpillar approach in his school, where 'what Ofsted are looking for' is used to justify many developments in the school. Finally, Luke goes back to a "*more traditional way"* which he described as "*balancing, keeping the equals signs in line"*. The absence of the pens in a pot in his "*more traditional way*" is particularly interesting, demonstrating that the enactive representation has been dropped in the devlopment of the use of the symbol as the 'unknown value' in an equation. Luke's classes have not derived relationships like 4c+5=81 represented as; "*four boxes of pens and five pens gives a total of 81 pens. How many pens in one box"*. This approach would have complemented the representation that he chose to use to develop pupils' understanding of variables in expressions demonstrated earlier, but was not valued in the context of solving equations. This reveals Luke's tendency to dabble with an enactive representation, but not to navigate the transition towards symbolic representations by allowing his pupils to truly make sense of the model (Bruner, 2006a). Luke is aware that he is still learning to teach and that he is reflecting on his experience.

> I think I needed to make the mistake first. Do what you want to do first. It might take a few years but over time you will figure out what's best for you. The cube method might not work for everybody. I'm glad of making the mistake. (Luke)

Again, this exposes Luke's tendency to focus on what the teacher is doing and what is best for him, in contrast with Ben's focus on what is best for the pupils. Luke is open to improving his practice once his first instincts for how to teach a topic, "*do what you want to* *do first"*, are unsuccessful. I asked Luke whether some of the criticality demonstrated in the above example was derived from the PGCE.

> I completely agree- the PGCE was really, really useful, there's some things that didn't necessarily work. I had my room in groups last year, this year they are in rows and the learning is much more effective. That was something that's nice in theory but in practice I've chosen something else. Another one for you, for place value. I use the chair model, I get them moving so that the place value is the size of the number. But I've found modelling that in a book is really difficult. Drawing out the chairs. So when they've come to do 0.065x1000 drawing out the chairs is quite difficult. What I've done is use the model, use the chairs so they've got their understanding, but when they go to their books, let them move the decimal point. So unless someone's come up with something better. (Ben)

I do not recall advocating that the classroom should be arranged in groups, but, nevertheless, this is the message that Luke has received when I have modelled the principles of a more connected classroom. Luke's description of his use of the chair model for teaching place value illustrates his partial application of a model developed in a university session. Once again, this reveals his enthusiasm for enactive approaches initially, but the model is quickly dropped in favour of a procedural approach when pupils are working in their note books. We talked about the enactive, iconic and symbolic stages of learning in his example, focussing particularly on the absence of the iconic stage in the lessons that Luke was describing.

> I agree with that, there needs to be a model there, but we need to find a decent way of putting that model into practice. Because if they can't do it in everyday life if they can't do this, I hate to say this in an exam, what's the point in doing it? They should have both, the understanding initially, but realistically, if they can take that understanding and show… this is what it is. (Luke)

Luke is describing the pupils' ability to reason how a process works, but also their fluency in multiplying and dividing by powers of 10. I agree with Luke, conceptual development and fluency are both important and his comments about the failure of transmission approaches suggest that he understands that fluency cannot be achieved without conceptual understanding. However, the enactive stage that Luke is introducing is likely to have little or no impact, or, even, negative impact, on learning unless he carefully guides the pupils through the transition from enactive to iconic to symbolic understanding by allowing the conditions for the iconic stage to be a feature of his classroom. The conversation ended at this unresolved point. Later that day, I received an email from Luke:

> You'll be pleased to know that after discussing the issue of multiplying and dividing by 10^x with [two experienced teachers in my department], that I will not be moving the decimal point and will stick to the 'chair' model. Thought you'd like to know. (Email from Luke)

Undeniably, I did like to know that he had reflected on our conversation, discussed his pupils' learning with other teachers and had planned to modify his practice.

Ben and Luke represent two perspectives on university-based mathematics education; Ben's complete acceptance that a connected approach is ideal, "*that's it, that's what everyone should do"* contrasted with Luke's acceptance that some of the ideas were really useful, but some did not work. As an NQT, Ben, is completely focussed on the experience of the learner, whereas Luke alludes to the experience of the learner, while focussing on what he does to "*prove/demonstrate/show*" concepts to the pupils. Ben accepts, unquestionably, that the models presented in university sessions are valid, because of his experience as a learner and his desire for his pupils to gain a relational understanding; applying methods that allow pupils to reason from key facts and make connections with complete trust in the effectiveness of the method. Luke's approach is less certain, dabbling with approaches that he is learning to value that are centred on the experience of the learners, while wrestling with his beliefs about his role as the builder and mender of his pupils' mathematical insight.

In Ben, I have always perceived complete trust in what I am teaching him and I have enjoyed his ability to challenge and stretch representations that we have used in university sessions. Luke approached the physical and practical models that I taught him with equal enthusiasm, but did not always engage in critical debate about the application of the models to their classroom contexts and the careful transition towards mathematical fluency. I am captivated by the absence of acknowledgement of my involvement in subsequent conversations with Luke. He read 'Ban the Equals Sign' because I gave him the article to inform his practitioner enquiry module. His classroom walls are adorned with activities to build pupils' resilience that I introduced him to. He came, with me, into Year 6 and 7 classrooms to explore the effectiveness of the 'sweets in a packet' or 'cubes in a box' method for building and interpreting expressions and linear equations. But this is not acknowledged in Luke's interviews, just as he did not acknowledge the origin of his development foci in his reflective writing during the PGCE.

I am not aggrieved that my contribution has not been acknowledged, but extremely curious about why. All of the events that Luke described took place in his classroom, he was the teacher and he helped the learning occur. I am not part of that world physically, nor does it seem, methodologically, because my role is situated within the university community of practitioners. Occasionally, I visit the world of school and the mathematics department, but this visit is a novelty. Any criticality that I may be able to offer Luke is situated away from the location of praxis, unlike the teachers cited in his email above who are trusted members of the daily community that Luke operates in. Ideas and approaches may be stimulated from or originate in university sessions, but they are enacted within Luke's school. Pens in a box is Luke's conception of an effective teaching model, because he chose to use it, developed it and applied it in his classroom. The origin of the model appears to be irrelevant in *his* classroom.

Ben appears to have maintained his relentless focus on his pupils' learning during his NQT year. His explanations suggest a classroom where learning is built on the contribution of the teacher and the learners. In this respect his classroom has characteristics of the democratic model described by Dewey (1916), whilst at the same time seeking enactive and iconic representations of mathematics to support his pupils' conceptual development. Ben's comments suggest that he is starting to articulate generalised principles to explain his pedagogy so that pupils "*build [concepts] up from basic principles*" and pupils "*enjoy seeing how those formulas were derived*". Multiple representations of mathematics concepts (Bruner, 2006a; Skemp, 1993) appear to be an evolving part of Ben's practice because he wants his pupils to understand mathematics and enjoy learning mathematics more successfully than his peers did when he was at school.

Both Ben and Luke have demonstrated traits of the connectionist teacher orientation in their explanations of their classrooms (Swan, 2005), and both describe the difficulties associated with learning mathematics that they have encountered in their practice. However, at this stage it appears that Ben is more sensitised to how his pupils learn, while Luke's descriptions suggest that it is his decisions that drive the activity in the classroom. In this respect, ben appears to be adopting an experience model of education more readily than Luke. This does not remove Ben from the teaching, his pupils are not left to discover mathematics for themselves, but he is able to choose the stimuli that he thinks his pupils will respond to as they construct meaning together (Dewey, 1938).

Chapter 7 Episode 4: Souvlakia and Turkey Twizzlers

This final episode is focussed on Anita's story. Anita is the school based mentor and previous PGCE student who first introduced the gateaux metaphor into my research. Originally, it was not my intention to include a school based mentor in this study, but as my ethnography evolved, I became conscious that this analysis included assumptions about Anita's intended meaning in sharing the gateaux metaphor. In order to expand my understanding of Anita's intended message, I interviewed her at the end of her first year as a school-based mentor to both a traditional PGCE and School Direct PGCE student. In setting the context for this story, I have included analogies made by other PGCE Mathematics tutors, as well as some of the students who have experienced support from Anita in school.

I shared the "Gateaux and one of your five-a-day" story discussed in Episode 1 with some other PGCE Mathematics tutors. One tutor suggested that 'gateaux' implies unnecessary indulgence; we can live without 'gateaux'. Together, we played with food metaphor until he came up with 'Turkey Twizzlers': the transmission lessons were not fruit, something natural and unspoilt that everyone needs in their diet. They were, in fact, something masquerading as food, something with some nutritional value, but much of it has been lost in the manufacturing process that has led to convenient, cheap and fast food like 'Turkey Twizzlers'. I am not teaching my students to heat and serve 'Turkey Twizzlers', but trying to help them understand how meals are prepared from fresh and natural ingredients.

A friend taught me how to cook Souvlakia, chicken skewers, which he described as Greece's healthy fast food. If my students can understand the recipe for cooking Souvlakia and I teach them the principles behind the balance of the flavours in the marinade, the timing and method of cooking, then they have the potential to provide their pupils with a more balanced, nutritious diet. With time, preparation becomes quicker and easier, perhaps having combinations of spices ready for use, because they have prepared the dish before. More importantly, they can understand the principles behind the recipe, so that they can adapt the ingredients and flavours to suit the needs of their pupils.

I ask my students to do something that is more difficult and more time consuming than heating Turkey Twizzlers. Something that is built on a belief that Turkey Twizzlers are served as a last resort, to be resorted to when there are no fresh ingredients available or no time to prepare the meal. If a teacher tries to change the diet that the pupils have become accustomed to, the pupils may be suspicious or resistant to change. I challenge my students to re-educate themselves and re-educate their pupils. Using Rachel's words from Episode 1, she has served 'Turkey Twizzlers' to her pupils because, "*it sounds awful, but it's just easier*".

I liken the fast and convenient diet served in transmission teaching to Skemp's model of instrumental understanding in mathematics (1976), an exclusive diet of instrumental mathematics. On the other hand, relational understanding is gained through the patient development of reasoning and conceptual understanding. I do not claim that relational understanding in a connectionist classroom (Swan, 2005, 2014) is easy, but I do believe that it is an accessible goal for my student teachers, who all claim to abhor the principles behind the transmission model when they are recruited to the PGCE.

I suspected that the PGCE tutor's interpretation of the 'gateaux' metaphor was not the same as Anita's because of what I had seen Anita teach as a PGCE student and how I had observed my students teaching in her classroom while she was their school-based mentor. Anita's concept of a '*five-a-day lesson'* was probably not a model masquerading as learning in the way that the '*Turkey Twizzlers'* were masquerading as food, but to the PGCE tutors that I had spoken to, the daily diet of transmission teaching observed in many classrooms could not be likened to something as nutritious as fruit. In my interview with Anita, I reminded her about the 'gateaux' presentation and the subsequent reactions of some of the PGCE students. I told Anita about the conversation around using an identity like $3x(2x+7)=6x^2+21x$ to stimulate discussions by asking pupils if what the teacher had written was true or false, or to convince the teacher in as many ways as they can that the equivalence of the expressions is true. Her immediate question was "*They interpret that as gateaux? So have I completely messed up your students minds?*". I assured her that I thought it was a great comment and that it had given us a metaphor to stimulate discussion about classroom practice.

> Genuinely, for that, its basic everyday teaching, it doesn't require planning … well planning… it does, but not '*planning planning'* if you know what I mean, it's just part of your teaching. If you're teaching [anything]… its just basic. That is basic.
> That's constantly part of your everyday lesson …to me that's not gateaux. Gateaux, for me, is if you are wanting them to do more of an investigative… like an open enquiry… something that you think you are going to have to structure very, very neatly and you're going to have to put a lot of planning into it to really get what you want out from it. (Anita)

Clearly, there was a difference in my colleague and Anita's interpretation of a Gateaux lesson. To Anita, asking questions with the aim of pupils justifying and making connections was "*just basic…. Constantly part of your everyday lesson*". Anita elaborated on her "*planning the structure very, very neatly"*:

> I think that's what happened to Jayne. She genuinely struggled with that whole reining it back in- you've investigated and …and when I was explaining it to her…. Even when it was an open-middle task, she was still trying to structure the task, in the sense that she planned the sort of direction that she was trying to massage them towards. (Anita)

Jayne, the student teacher that Anita was working with, was trying to include enquiry in her teaching, but was trying to plan for every eventuality by attempting to structure the task so that the she could *massage* the pupils in the direction that she wanted. I do teach my students a model of enquiry that we refer to as 'open-middled', rather than the more widely used term 'open-ended', because I teach them to provide a stimulus for learning, to give the pupils freedom within an 'open middle' to make sense of the stimulus, which would usually converge on a goal.

Mathematics is built on agreed truths, a universal goal, but the way that each learners' mind makes sense of that goal is arbitrary (Wenger, 2009). A mathematics teacher can prescribe the truthfulness of a statement like '37 is prime' but cannot prescribe the way that the learner's mind conceives the primeness of 37. In that respect, the goal of teaching might be to know that 37 is prime and to know why. Yet the 'open middle' is a metaphor for the subjectivity inherent in each learner's cognition and the arbitrary way each learner responds to the learning stimulus. Anita's description of our student teacher's struggle to build on pupils' ideas, in order to converge on a goal, illustrates the difficulties associated with fostering relational understanding at the early stages of teacher development.

In some, albeit limited respects, Anita is adopting Dewey's model of a more democratic classroom fostered through learners' experience (1938), whereby she tries to respond to the students' observations and questions. She illustrated this in the following description:

> Ian the other day… this is quite nice for him.. He said to the class 'what is enlargement?', a child said 'making the shape bigger' and he said 'yes that's right'. And then it dawned on him… From a teaching and learning perspective has never thought about [enlargement making objects smaller having the potential to confuse pupils] before. When I was saying this to him it was obvious, but at first he was unsure what to do. And then when we talked about his questioning, I said you could have opened up a lovely discussion, is it always bigger? What are the conditions to make it smaller? He could have gone down so many different routes and part of that will come from confidence in teaching and maybe at some point a student has prompted me to think about it when I was training. (Anita)

Ian knew how to relate the concept of enlargement to objects and images and how to solve problems within that concept. His own knowledge of mathematics was very good, but his mathematical pedagogical knowledge was being stretched by this incident (Shulman, 1987). He had not thought about the disturbance that the everyday use of the term enlargement could create in the context of the mathematical interpretation of the term. He had not stopped to think about this because, until this lesson, when "*it dawned on him"* he was not thinking about the experiences that the pupils would bring to the lesson. Ian had attended university sessions on planning lessons that recognise the complexity of language in the mathematics classroom by explicitly teaching mathematics terms and modelling their use in context. However, until he faced this issue in practice he was unaware that the use of the mathematical term ‘enlargement’ was an issue at all.

The pupil responded to Ian’s question with a common sense interpretation of “*what is enlargement*?”, rather than the potentially counter intuitive mathematical response suggested by Anita, prompting Ian to question how his pupils would learn the conditions for an enlargement scale factor to make the shape smaller or to alter the orientation of the shape. Jayne, who Anita described as ‘massaging’ the students towards a goal was trying to structure every part of her lesson in the guise of an enquiry approach; an undemocratic approach that assumes that her conceived route through the problem is the ideal route for the pupils to follow. Jayne had planned a solution to the problem in minute detail and so struggled to teach, to “*rein it back in”* when the pupils had followed different routes through the problem. However, Ian was attempting to respond to the pupils’ sensible suggestion that enlargement makes objects bigger, but did so inexpertly. Anita’s suggestions for questions posed represents a more democratic approach because it is building on the learner’s knowledge and using their powers to construct meaning (Dewey, 1938). She does not specify which experiences the pupils were using to stimulate their learning, but she does imply that she is open to pupils’ suggestions because there are “*so many different routes to go down”*. She is aware that experience, leading towards a more expert teaching approach will give her student teachers the confidence to follow those different routes, acknowledging that she had probably thought deeply about this potentially confusing aspect of the language of geometry when “*a student has prompted me to think about it when I was training”*.

To illustrate the perceived demands of the pedagogy I teach my students, modelled on asking, rather than telling (Halmos, 1985) had been Gateaux to Kate, the PGCE student from the encounter in my office, but was basic teaching to Anita. To Kate, asking questions had only seemed obvious once I had suggested them. We discussed some examples of physical and practical activities that Anita and Kate had used in enrichment experiences in primary schools during the PGCE course.

> Yes, that's gateaux, not your [everyday classroom activities]… or Like Perigal's Dissection. If you really wanted to do it into an in depth… how far can you take it? Because some people would just do it as a starter and show [pupils how to] prove Pythagoras. Construction outside with the chalk, practical trigonometry, something I'm really going to have to think about. That, to me, is gateaux. An essential of fantastic teaching. (Anita)

A lesson that is built on a physical or practical experience requires a lot of planning and thinking and so, is Gateaux; something Anita had claimed was not needed every day, but remains "*an essential of fantastic teaching*". This means that "*fantastic teaching*" is something that she would encourage student teachers to aspire to from time to time, but that they should aspire to good teaching, with effective questions in a regular five-a-day lesson.

Anita and I seemed to share some similar approach to teaching beginning teachers, at least within the asking not telling principles revealed in this interview. Nevertheless, some of the students who heard Anita's presentation had received a different message. Anita started to rethink her comments to the PGCE students:

> Maybe on reflection, what I should have been saying to the PGCE students is that maybe at first, you will have to spend a long time planning a bit of Gateaux but as you plan more and more lessons you'll bring a little bit of Gateaux into your everyday teaching into your five-a-day. (Anita)

Anita described a recent lesson with a Year 7 class that had included a physical activity to interpret simple linear functions.

> [One] of those things that I thought 'I'm really happy I've come up with this one and I thought that's really good I'll use that again', but I hadn't planned to do it. But stuff like that, in hindsight there is an element of risk, that could have gone messy. If someone had walked in it could have looked like my kids just wandering around the room… but if you are confident and comfortable with the kids you're working with then [you can do that]…. But some people might have seen that as Gateaux but I just see that as… everyday teaching. (Anita)

This suggests that Anita recognises greater risk in physical or practical classroom activities. The activity "*could have gone messy. If someone had walked in, it could have looked like"* alludes to her acknowledgement that the physical activity was not a typical activity in the community that she works in. Was the messiness associated with the risk that pupils may have failed to learn from the activity, or because she may have appeared to have lost control of her pupils?

Anita's claim that "*if you are confident and comfortable"* with the pupils that you are working with, you can subsequently take more risks with activities in your classroom is revealing. The beginning teachers that Anita was working with were not necessarily confident that they had trusting relationships with their pupils nor were they comfortable with the context for learning that currently existed in their classrooms. Anita could understand how student teachers were not always teaching in a context where they could take risks:

> I completely understand why they view [asking rather than telling] as risky because while you are training you are always in a class with someone else. And I suppose when you are in your class just with the students, if that avenue went so messy you could say to the kids I'll come back to you on that one, and you don't feel judged… but if you're being watched… maybe with the observer, [the student teacher is wondering] how would they approach this? But, you've got that thirst from the kids when they're young… they want to know what it means. But… sometimes…I do think it often comes down to time. Some [student teachers] think that putting 'true and false' on the board does take time. (Anita)

Anita offers two further reasons for the student teachers' reluctance to ask questions classroom. The first is the recurring theme of perceived time constraints, suggesting that telling is perceived as quicker than asking. The second is 'being watched'. Student teachers' are subject to on-going assessment in their regular lesson observations and feedback sessions and, as with any form of assessment, there will be an emotional response attached to the assessment (Black & Wiliam, 1998). Anita alluded to her own fear that "*if someone had walked in it could have looked like my kids just wandering around the room*" when she was using a physical activity in her teaching. The student teachers that she supports are constantly being watched and, therefore, potentially share her apprehension that riskier activities may appear as though the student teacher does not have control of the class.

This leaves the novice teacher with two potential barriers to a more connected classroom, the threat of losing control of the class, because they do not yet have trusting relationships between themselves and their pupils and the dilemma of potential '*messiness'* due to inexperienced approaches to dialogue, because they have not yet thought about, or been prompted to think about potential connections and disturbances within mathematics. Anita had captured this splendidly; with respect for the context for learning, higher risk activities are more readily attempted if "*you are confident and comfortable with the kids you're working with*" and, with respect to the construction of meaning in the classroom going "*down so many different routes"* stimulated by the pupils' responses will come from *"the confidence in teaching*" that comes with experience. In this respect, Anita is describing aspects of teacher knowledge described by Shulman (1986, 1987). The beginning teacher's knowledge of the context for learning can only come from the school-based education, so that immersion in the culture of that school leads to confidence and comfort in working with the pupils who are also immersed in that culture. The locus of the student teacher's development of mathematical pedagogical knowledge is less clear. University teacher education can give Jayne and Ian an experience based model of education, but they can only practise this model in school, in a place where an experience model may conflict with the school culture.

Anita empathised with the student teachers' dilemma:

> But then in some ways [when the student teachers do not see connections in the structure of mathematics] you think how limiting, because once they've made that connection… But when you're training its information overload isn't it. And that's what I said to Jayne that you're enriching them with so many ideas and it's really good, but they might think its black and white and they have to do everything in every lesson. And if they struggle because they're stressing about behaviour management or planning… I did spend three hours in a starter on the PGCE and part is having no resource and part of that is trying to follow a principle. (Anita)

I agree that a student teacher may experience "*information overload*" during the intense 10 month period in which they train to teach. If Anita really did spend three hours planning an activity to start her lesson in her first PGCE teaching practice, there is clearly a gap between the experience modelled at university and her practice in school. Perhaps "*having no resource*" suggests that she had no resource other than textbooks and worksheets, which are usually readily available in school. It is possible that Anita's perception that the activities modelled in university are rich, led her to seek rich activities. However, I maintain that the models provided in the induction period do not offer a model of rich resources, but rich learning experiences because of the questions asked and the way that mathematics is presented in a learnable manner, allowing pupils to make connections and to reason mathematically. This was exemplified in the area of a triangle experiences discussed in previous episodes. Richness comes from making connections with the area of the rectangle and not in the design of a resource. Practice questions from any text book or worksheet could be used to teach pupils how to find the area of a triangle, but whether pupils make sense of the area as 'half a rectangle' is dependent on the questions that the teacher asks and the way that the teacher responds to the pupils' answers. It is the activity of the teacher that determines whether the concept is accessible to the pupils because pedagogy is fundamental to learning in school. Anita is right that "*trying to follow a principle*" can present a barrier to the beginning teachers fostering a connected classroom, but I do not agree that lack of resource is the issue.

In the case of Ian's experience in the classroom, there should be little perception of ambiguous pedagogy because Anita and I share a similar belief about dialogue in the classroom and model a similar approach that builds on pupils' powers. However, Anita describes the situation in her mathematics department, suggesting that her beliefs and practices are not typical of the culture in the department as a whole.

The constraint of time was a limiting factor cited in Anita's own teaching, as well as in her account of her student teachers' experiences.

> I do take risks in my teaching. I think sometimes the [scheme of work] and the pressure that you're under and the amount you're trying to get through… I think it minimises sometimes how much time you have to plan creatively. I think the way the system has gone is that you spend so much time trying to mark and get the marking right for Ofsted, because they want it in a particular way or the school wants it in a particular way. That sometimes overshadows your planning time. (Anita)

The pressures listed here ran through Anita's account of her influence as a Key Stage Co-ordinator in her department. We talked about features of Anita's practice that were aligned with the connectionist teacher orientation. I asked her whether her beliefs were shared by other people in the department.

> I suppose in some ways that if that's not happening and … There's been loads of conversations so far when I've made suggestions that 'this is how I teach it' and teachers who have been stuck in their ways for years have said 'oh I liked the way you did that can I have a copy of it'. Which is fantastic….But some people who have been in the profession for 25 years are not willing for someone who's been in for 2 years to explain it to them. They don't see it as this is new kind of blood. But [the Head of Mathematics] at [my second placement school] says one of his reasons for having PGCE students is just the right sort of reason 'they are the perfect CPD for us because they are the ones who have their finger in the pulse who are going through their training and the training is freshest and they will bring in ideas and help support the department that way' and that's a really good way to do it. But I've had comments from people in the department … well you've only been teaching for two or three years. (Anita)

How did Anita explain differences between her beliefs and those of her colleagues?

> That comes from experience and your ethos. It's your mind-set… you look for connections. Once you have that approach to your teaching then you constantly come up with these ideas and it's not just like you've got a bible that you're following. (Anita)

Anita is right. A connectionist orientation that fosters relational understanding does not come with a manual or "*bible"*, but is based on a teaching philosophy relating to how mathematics is learned and what mathematics is. Mason's theory of the Discipline of Noticing (2005) supports Anita's claim that once a teacher perceives the benefits of a more connected approach to learning, then they become disciplined to notice situations where a relational understanding can be fostered, they become attuned to experiences that expose the impact of relational understanding and the shortcomings of instrumental understanding and, through this, develop a wider array of experiences on which to draw in situations like the conflict about enlargement described earlier.

However, Anita's learning trajectory has been influenced by encounters that have exposed her to the value of a more connected pedagogy (Wenger, 2009), in a manner that some of the colleagues that she describes may have not. I agree that her colleagues' dispositions towards learning and beliefs about learning influence their behaviour in the classroom, but they have also been subject to different encounters in relation to their professional learning. Anita gave examples of teachers avoiding the aspects of a concept that is likely to cause disturbance in their classrooms.

> You see that a lot actually. A conversation that I had when we were putting up a display board for open evening. And I used factor trees and I wrote up the heading *Prime Factor Decomposition* and the teacher said 'You're putting up the word decomposition up on the board?' and I said yeah because that's what it's called, but it's that idea that it's too hard for the kids to understand and that idea of dumbing it down… my Year 8s can understand decomposition. [Some teachers] dumb it down rather than teach it. Like, my year 11 now are still trying to understand the language and I want to expose them to it from the beginning. (Anita)

This demonstrates Anita's belief that mathematical understanding is developed through language and dialogue, which is at the heart of the connectionist teacher orientation (Askew, 2002). She also recognised that a relational understanding of mathematics takes longer initially, but has characteristics of mastery learning so that concepts are not being retaught year after year.

> There's no patience sometimes… [The teachers] don't have patience and sometimes you have to view it as an investment. They are drilled [that] every single lesson has to have pace but if you view your lessons from front to back as a series, as a chapter of learning rather than just a lesson- you might go a bit slower at the beginning and the pace will come at the end which will probably be much steeper learning at the end. (Anita)

Anita suggests that teachers lack patience, which is how it may appear to her in a culture where she perceives that her school leaders expect to see lessons with pace, where learning is evidenced in each lesson and the learning occurs quickly. Evidently, this conflicts with Anita's beliefs about how her pupils learn, but it also reinforces her colleagues' beliefs that a more connected pedagogy is incompatible with the context in which they teach. These teachers work in a neoliberal culture, through which the signifiers of effective teaching (pace, examination results, visible marking feedback, programme coverage) are aligned to the characteristics of the transmission teacher orientation (Ball, 2013) more closely that the connectionist one.

She elaborated further on the notion of pace.

> When we bought in the [new award for Year 9 pupils] and teachers were saying to me 8 weeks is way too long and I was thinking, what are you doing? How can that be possible? When you think all of the depth that's in that. A naive side of me thought 'Are your kids super and they've just absorbed everything?', but actually they've just done the peanut… or whatever method. The whole point was that was an amazing opportunity that we can just spend 8 weeks looking at the fundamentals of number and measure that they've never grasped and they've just whizzed through it in the normal way that we teach. They're not thinking about learning… and I'm thinking about learning and if I can get my Year 9s learning this really solidly, then year 11 is going to be a doddle. And I've found out that some of mine… they know just to explore it a little and for enlargement they said we've kind of realised that if we go that way for positive, we go back here for negative. It was amazing… you start to realise that the kids take on the philosophy and it makes your life so much easier… if you have the patience for it…because you're not teaching everything and they're learning. (Anita)

Using a connectionist orientation, Anita has created a culture of connection-making, so that her pupils are no longer resistant to the change of culture in her classroom. Anita believes that her pupils making connections and moving between representations makes her "*life so much easier*". The transmission orientated teacher is covering every mathematical possibility within a concept, transferring their conception of a concept to their pupils so that the teacher is doing most of the mathematical thinking and the pupils are receiving the teacher's pre-ordained conception of the concept. In this respect, transmission is hard work (Boaler, 2009). An instrumental understanding is hard work because of the absence of connections and the improbability that learners will apply knowledge independently.

The "*peanut method*" has become a metaphor in several university sessions for classrooms built on a procedural method designed to guarantee two marks in an exam, irrespective of the absence of meaning behind adding and subtracting fractions and the restriction of thinking about the equivalence of fractions to adding and subtracting exclusively. The peanut method, in my perception, was limiting the pupils' experience of fractions and restricting their ability to reason and make connections, as well as limiting their potential to work assuredly with fractions in algebraic contexts. Some of my students had justified the use of the peanut method for adding fractions for GCSE pupils whose exams were fast approaching and who still hadn't mastered addition and subtraction of fractions. Anita could see some strength in their justification, but could not accept this in her own practice.

> Then having said that with the adding fractions when I first took on my year 11s and some of them were on Us and Es and I'm still trying to push them closer and closer to getting a C and even when they were doing adding fractions at the start of the year it was still sort of questioned [by a colleague] why would you go to the depths of common denominators and equivalent fractions and unpicking it that way… Why would you not just draw a grid and multiply across and I thought that even at this stage… I thought if they can actually do it with understanding… that's fundamental… People do [the peanut method] for speed. (Anita)

Speed, once again, forms part of Anita's justification for teacher's adopting a procedural context in mathematics classrooms. It is possible that her teachers believe that they would be found wanting by restricting curriculum coverage for their pupils; living with the terror of performativity described by Ball (2013). But Anita had already justified the ill-conceived notion that this approach is quicker, because viewing lessons as "*a chapter of learning rather than just a lesson, you might go a bit slower at the beginning and the pace will come at the end, which will probably be much steeper learning at the end*". The speed excuse probably masks teachers' reluctance to tackle the complexity of building a classroom dominated by relational understanding, because of the disturbance that this would cause to the culture of learning in their schools and because of their own conscious or subconscious awareness that Anita's mathematical pedagogical insight exceeds their own. Anita suggests that teachers' lack the patience needed to build a classroom where experiences allow pupils to make connections and reason mathematically, but it is so much more than patience that is needed, not only to see the connections themselves, but to know how to present the mathematics in a manner that allows pupils to make those connections too. Anita's description of her lessons sound as though they are full of activity, she is not referring to the pace of activity in her classroom, but is actually referring to the pace at which teachers 'cover' the curriculum. Darling-Hammond (2010) describes this approach to learning mathematics as a mile wide, but only an inch deep and recognises these characteristics in classrooms in the United Kingdom and USA. She cites countries that have more successful mathematics learning because teachers foster deeper understanding earlier in pupils' education, where understanding dictates the time available to learn a concept and not the time allocated on a schools' learning programme. This is the phenomenon that Anita is describing. The mile wide and inch deep approach to teaching is the consequence of a diet of 'turkey twizzlers'; fast, ready-made definitions, shared using teachers preconceived definitions of concepts that pupils are required to reproduce, rather than understand. However, Anita is trying to develop a more nutritious, balanced diet that takes longer to develop but nourishes pupils' understanding and mathematical thinking.

For most of my students, they are not explicitly told which method to teach in their lessons. In most cases, it is up to the student to choose the approach that they take, although they report implied pressure to follow a particular method through comments such as "*I wouldn't do that it takes too long*" or "*they don't need that, they're set 1*". Clearly, it would take a lot of tenacity for a beginning teacher to ignore these messages. One student, Ruth had planned a classroom enquiry designed to develop Year 7 pupils' collaborative learning in her classroom as part of her PGCE Practitioner Enquiry module. She included the following in her account of the enquiry:

> Adding and subtracting fractions required pupils to use previous knowledge of finding common denominators to understand the concept. I wanted them to see that when adding or subtracting fractions the denominators need to be the same. However, the class teacher asked me to concentrate on a different 'peanut method' of addition and subtraction for the next lesson. Although I was unhappy to be delivering a 'quick fix' method that built no understanding for the pupils and opposed my teaching philosophy, I followed the teacher's instructions to try the method out with the class. (Ruth)

She provided an insightful, research informed account of her pupils' transition from an inclination to adopt *disputation* talk to *exploratory* talk (Mercer, 1995), despite the "*quick fix*" method that she felt she had to use with the class. She concluded:

> I could have improved the intervention by setting pupils a rich problem solving task with practical resources to allow them to see for themselves when adding fractions the denominators have to be the same. I felt that, as predicted, although pupils could understand and use the 'peanut methods' successfully, they gained no understanding of how the method worked. (Ruth)

This experience increased Ruth's resolve to adopt a connected approach to teaching addition of fractions in her NQT year and probably taught her to comply with problematic requests from her head of department, until she is in a position to change policy. However, Ruth maintains some agency in the mathematics pedagogy that she adopts by choosing the school that she wants to work in for her NQT post. I chose this placement school for Ruth so that she would experience a context that contrasted with her first placement school. It is very likely that Ruth would not have accepted a post in this school, because of her awareness that the practices of the mathematics department "*opposed my teaching philosophy*". In Ruth's account, using the 'peanut method' is like feeding the pupils 'Turkey Twizzlers' and she is aware of the shortcomings of this approach but feels disempowered to use an alternative, relational approach in the context of her PGCE placement school.

Conversations with teachers like Anita and Ruth are cathartic. They both operate under the same political and policy constraints that teachers report in secondary schools (Ball, 2003; Furlong, 2013; Lerman, 2014), but both aspire to act on their beliefs in order to foster an environment where pupils are more likely to gain a relational understanding of mathematics. They are characterised by their certain beliefs about pedagogy, even though this has probably not been wholly translated into practice, into pedagogy yet. I am not looking for traces of my teaching in their practice, but I am looking for accounts of classrooms where learners, as Anita described: "*take on the philosophy and it makes your life so much easier… if you have the patience for it…because you're not teaching everything and they're learning*" (Anita). The pupils' development of relational understanding is Anita and Ruth's focus for development. They do not need to turn to me for resources and activities to teach every aspect of the secondary curriculum because they have learned a principle described by Ruth as: "*setting pupils a rich problem solving task with practical resources to allow them to see for themselves*". Pupils make connections between and within the structure of mathematics so that they may become fluent mathematicians, knowing how to solve problems and reasoning why procedures work. Anita describes this as a mind-set, an approach to teaching that is framed by a belief that a connected classroom is a more effective setting for learning mathematics than a transmission classroom. As she said, once the teacher has this belief, then this principle informs pedagogy, "*you look for connections. Once you have that approach to your teaching then you constantly come up with these ideas and it's not just like you've got a bible that you're following"* (Anita).

I do not have a manual to give to my student teachers, but I am able to justify my pedagogical approach and I model that pedagogy consistently at university and during conversations that relate to school observations. I cannot possibly teach my students how to teach every aspect of the curriculum that they are training to teach and nor would I want to. I want them to learn pedagogical principles and not lesson activities that can be replicated in their classrooms. I do share activities and resources, but this is my attempt to provide some elements of an experience based model of teacher education at the university. Naturally, I want them to adopt the research informed resources and approaches to teaching that I share with them, but moreover I want them to learn a pedagogical principle so that they may have some chance of adapting to their own classes and their own school contexts.

Generally, neither teacher is serving pupils a diet of "Turkey Twizzlers". They are rejecting 'quick-fix' models like DNO and 'the peanut method' because they do not believe in the value of instrumental knowledge without mathematical reasoning and insight into the connected structure of mathematics.

Anita's words are insightful and worth repeating: "*you look for connections. Once you have that approach to your teaching then you constantly come up with these ideas*". When my students are training in Anita's classroom, I feel confident that my student teachers receive less ambiguous messages about pedagogy than in some other mentors' classrooms. This episode exposed differences in interpretation of rich gateaux and five-a-day lessons, but also revealed our consistent beliefs about pedagogical principles such as asking and not telling or constructing an experience model that builds on pupils' powers. Yet even when a teacher like Ruth is placed in the difficult position of teaching in a manner that conflicts with what she believes, she is still able to hold onto her belief that pupils learn through rich questions and problem solving, including enactive learning models when required. I have seen these principles in practice when I have observed Ruth and Anita teaching and I have judged from the reactions of their pupils that relational understanding is being developed alongside mathematical fluency; they appear to be genuine beliefs that determine their actions in the classroom, rather than desirable versions of themselves presented to their PGCE tutor. I did not ask them to separate which aspects of their pedagogical principles they thought that they had learned from me because it is less important than the practice of the principles in secondary school mathematics classrooms. Once an action that has been stimulated by my teaching has been applied by my students, the action becomes theirs; thus, becoming distinctive in the contexts in which they have decided to act.

Chapter 8 Findings

The previous four chapters provide narrative accounts of some of my students' experiences of learning to teach. In each episode, the student and beginning teachers share their perceptions of their experiences of learning to teach and how this experience is influenced by university-based and school-based teacher educators. Throughout they reflect their beliefs about mathematics education and justify their perceptions of how their beliefs influence their actions in the secondary mathematics classroom. These features address my research questions and expose four themes that I analyse in greater depth in this chapter.

In all of the episodes, teachers describe their evolving professional knowledge and talk about shifting the focus of their attention from what they do as teachers to how their actions influence their learners, analysed below as a focus on teaching *or* learning. For each student, tensions exist between their beliefs about pedagogical principles and how these principles are complementary or clashing in the cultures in which they teach. This theme is analysed in the second section of this chapter through the lens of the theory of situated cognition in the school culture where my students teach. From my students accounts of the situated nature of their learning and their pupils' learning, issues arise that relate to the incompatibility of aspects of the pedagogy that they encounter in school or at university, which is discussed in the third section. The final section of this chapter summarises the features of their teacher education that facilitate or restrict their developing praxis; looking beyond their reflections on their practice to locate a space for teacher education that allows them to have agency in their developing roles.

8.1 Focus on Teaching or Learning

All of the qualifying or recently qualified mathematics teachers in this study revealed dispositions towards their roles that were dominated by their focus on the actions of their pupils directed towards learning or dominated by their own actions as teachers in directing the learning. Anna's comment succinctly exposes the tension apparent in this transition *"I'm a teacher who's still doing the doing, instead of letting my pupils do the learning"*. I have seen many of my PGCE students make a significant leap in their ability to plan and teach more effectively when they shift the focus of their planning from 'what am I doing as a teacher', to 'what are my pupils doing as learners'. Anna knew what she was trying to achieve in her pedagogical approach, *"There's still an element of me that wants to teach like that, [in a connected way]… an element that sees that's where I should be trying to get to"*. The '*still'* in her comment exposes the turmoil felt as she was trying to make the transition from focussing on what she was 'delivering' to the pupils towards a focus on understanding what and how her pupils were learning.

For other students, less turmoil was demonstrated. Sam explained her reasons for teaching sequences in a more connected manner*, "I think it took them longer to grasp what they were actually doing and why they were actually doing it, but I like to think, in the long term, it's going to stick with them a bit more"* and Rachel offered a similar justification, "*Rather than you just telling them how to do something they've done it by themselves, like independently"*. Sam's assertion was certain; "*That's what it's all about anyway, it should be about the learning and not about the teaching"*. But Rachel shared some of Anna's uncertainty. She recognised that transmission teaching is sometimes resorted to as an "*awful but easier*" approach to mathematics teaching. Recurring themes of time constraints and the 'messiness' of causing disturbances and promoting a relational understanding were apparent in all of the research data.

Luke's approach, in "The Caterpillar Method" episode was interesting because he described a range of approaches to teaching in his NQT year that were characteristic of transmission and connectionist orientations, but reported, with certainty, his belief that he was moving towards wanting the pupils to make connections independently.

> When I first started this year it was a case of I want to be…. and this is going to sound really egotistical now… I want to be at the front. I want them to take what I'm doing, listen to it and replicate it in another problem and set it out the way I've set it out. (Luke)

These statements seem to contradict Luke's actions in the lesson that I observed, where he exposed misconceptions in algebraic expressions, but erased the misconceptions without addressing them. Luke is dabbling with the connectionist orientation, but closing connections down by providing correct answers that dissipate the pupils' disturbance. It was okay, he claimed, for pupils to offer wrong answers in his class, but it was not okay for the pupils to try to resolve them. Luke offered a justification for this because "*What I've done in the past is brought up a misconception, tried to explain it and confused them even more. So what I thought I would do is prove to them that's right".* Luke was still a teacher who was '*doing the doing'*, experimenting with learning models that promote connections, but transmitting the reasoning process that robs the pupils of opportunities to deepen their understanding of the structure of mathematics. This was confirmed in my mind by his use of terms "*I will prove"* and "*I wanted to"* throughout his interview.

This contrasted with the approach of the other NQT, Ben, whose focus had been on the learning experience of the pupils in all of the lesson that I have seen him teach, "*It's all about the learners, that's what it's all about isn't it?*". Ben was determined to give his pupils experiences that he felt that he had missed out on as a secondary pupil in a transmission orientated classroom. Ben has a clear articulation of the contrast between transmission and connectionist orientations; "*I was taught maths in this room, and it was very, you know, these are the formulas get on with them and fortunately I just picked them up understood what it was. But there were still people in my class who just couldn't… why have you put those in why did you do that? It just makes sense, build it up from basic principles. Why anyone would do any different?*". Whilst building on key known facts is not the only feature of a connected classroom, his question is discerning, why would anyone adopt anything other than a connectionist orientation?

Rachel suggests that it is easier to use transmission teaching, which may be a misconception. A transmission orientated teacher is doing all of the work, conceiving the mathematics to be transferred, explaining and answering. Whereas a connectionist orientated teacher is allowing the pupils make sense of problems, to reason and provide solutions (Swan, 2005). Anita, the mentor in Episode 4, understood this, "*you start to realise that the kids take on the philosophy and it makes your life so much easier… if you have the patience for it…because you're not teaching everything and they're learning".* Nonetheless, at Rachel's stage in her teacher education she believes that telling is easier than asking, even if it is an orientation that she does not value because "*it sounds awful*".

So, the ‘it’s easier’ argument may be rooted in more complicated reasons, in that connectionist orientated teaching assumes that understanding is built on prior knowledge, key known facts, and that misconceptions are exposed because of the insight they give into the structure of mathematics; providing a stimulus for reasoning and understanding the structure of the true concept. These factors expose the ‘messiness’ of pupils building schema because of the inherent subjectivity within each pupil’s mathematical understanding. Each child makes links within a network of knowledge in his or her own way, which does not fit within the linear, progressive simplicity of the transmission teacher’s beliefs about mathematics (Swan, 2005). Each child’s knowledge is situated within the culture and context in which the knowledge is built and is subject to their individual predispositions (Bruner, 1996; Brown, J. S., 1988). In assuming the connectionist orientation, the teacher is relinquishing control of how a mathematical concept might be conceived; the teacher is embarking on a socially constructed orientation that allows for all parties to contribute to the learning process (Dewey, 1916; 1933; 1938). The connectionist classroom is a more democratic place to learn, where knowledge is built through the experience of the learner (Dewey, 1938) and not by the replication of the pre-conceived knowledge of the teacher.

Further to the complexity of building mathematical knowledge is the issue of time and pressures such as assessment and marking that distract teachers from developing their practice. This was discussed by almost all of the participants in this study. Adam questioned why “*the machinery*” of teacher education was not doing more to interrogate why retention rates in teaching are worse than many other professions. I am not sure that teacher educators *can* do more to alleviate this problem. Ball describes how "*the technologies of reform produce new kinds of teacher subjects"* (2003, p. 217) created by the performativity culture that permeates education. My student teachers are learning to teach within this climate, where they "*are subject to a myriad of judgements, measures, comparisons and targets*" (Ball, 2013, p.8). Through this they are continually accountable and constantly judged so that to risk visibly teaching in a manner that is not valued as demonstrating 'what matters' by those measuring and judging the teachers' performance is an enormous risk for all teachers.

In my university sessions, I model a connectionist approach that, amongst other things, exposes and redresses misconceptions by allowing pupils to articulate a conflict and then presents them with information from other pupils or the teacher, which allows them to resolve the conflict because the new information becomes incompatible with the misconception; incompatible with their earlier beliefs about the concept. The pupils may then learn from the conflict because they have adapted to the new information, or they may not, because working through the disturbance caused by the conflict presents too much difficulty for them, resulting in responses such as disorientation or rejection of the new information and reverting back to the misconceived prior learning (Skemp, 1993). This same model is apparent in my students' teacher education. My pedagogy, perhaps modelled through resources or activities may disturb them and possibly conflict with the dominant pedagogy that they are seeing in school. Anna expressed this difference; "…*we've all questioned ourselves, we've all come back and said 'why is it we're not seeing what we're talking about here in schools?' and you know… we're wiser now to know that it's because* *teachers tend to fall into the easy category and teaching connected is not easy*". So my students have to work through the disturbance caused by the conflict. They can, like the pupils learning mathematics, seek new evidence that allows a connectionist model to work in their school contexts, or they can justify rejecting the connectionist model, 'that won't work here because it takes too long' or they can reduce the significance of the connectionist model 'they're nice activities but the pupils just want me to tell them'. Whether it is the student learning to teach or the pupil learning mathematics, this demonstrates that assimilation of knowledge, whether conceived or misconceived, is easier than accommodation (Skemp, 1993).

I am without doubt that assuming a connectionist orientation as an early model of practice results in beginning teachers shifting their focus from their behaviour to the experience of the learners more readily than a transmission orientation. However, learning to teach is situated in the culture of the secondary schools that they train in, where the reinforcing influence of the university teacher education, of the dominantly connectionist model, is usually absent. I share compelling evidence that the pedagogy I teach my students has the potential to build their pupils' understanding, but if this pedagogy is not conceived as compatible with the criteria by which teachers are judged, my students have little agency in developing their professional knowledge in the manner that they perceive from university teacher education sessions.

8.2 Situated Cognition in a School Culture

Learning to become a mathematics teacher in a secondary school is, for all of the students that I teach, situated in the culture and context of the schools and mathematics departments that they teach in. My teaching is situated away from this context and represents a separate culture of university teacher education, within a community of practitioners that is not necessarily representative of the school. Adam's articulation of me signifying a bridge between the two sites where learning is situated illuminates his perceptions of the separation between university and school mathematics teacher education.

Brown and colleagues made a significant contribution to understanding the relationship between the culture for learning, the activity surrounding the learning and what is learned. They concluded that "*the unheralded importance of activity and enculturation to learning suggests that much common educational practice is the victim of an inadequate epistemology*" (J. S. Brown et. al., 1989, p. 35). Earlier, I argued that I offer my students a model of pedagogy that I reason by justifying my pedagogical and epistemological assumptions, which I relate to classroom contexts derived from my experience. Models of teaching mathematics offered in school do not necessarily offer ambiguous pedagogy, but do suggest ambiguous epistemology. Anita, for example, offers the student teachers that she mentors clear justifications for her pedagogy as seen in her account of effective questioning as "*just basic*" teaching, and if I had given her the opportunity, she may also have been able to justify her epistemological assumptions. But, as Anita suggests, her model is not the dominant approach in the mathematics department in which she teaches. This is a view confirmed by most of my student teachers as well as Ofsted inspectors (2008, 2012). This situation leaves my student teachers with two dilemmas. The first is the "*inadequate epistemology*" (Brown, et al, 2008) that underpins the proliferation of transmission orientated mathematics teaching. The second is the difficulty encountered when a pedagogy underpinned by a more adequate epistemology is situated away from the 'activity and enculturation' where learning to teach mathematics is enacted. My main finding in this study encapsulates the second dilemma.

In "Episode 2 Divorcing Parents", Adam described the mathematics teachers having "*high regard*" for me in his department as the nub of the problem in his experience of teacher education:

> Therein lies the problem, they can preface their comment with "Sally's great at what she does". Then you know there's a *but* coming. […] The one thing we must guard against is the university tutor coming along and then leaves and everyone has a pop. (Adam)

What exactly am I great at? Does this mean that I am "*great*" at the teaching that *I* do, university teaching, but that this teaching is not related to the mathematics teachers in his department? "*Sally's great at what she does*" is not "*Sally's a great teacher*". Adam is absolutely right; this is exactly the source of difficulty in my students' teacher education. What I do is not situated in a secondary school classroom. The responsibility for the pupils' learning in Adam's school does not lie with me. I may be "*great at what I do*", but that is not secondary mathematics teaching. Adam seems to know that my presence in the "*world of school*" causes a disturbance because "*there's a but coming*" and that this disturbance is ratified by teachers in his department reducing the significance of what my pedagogical models represent, by taking "*a pop*".

It is no surprise that this situation causes Adam difficulty. He has grown to respect what I teach him because he has *"complete trust and confidence that what [I am] explaining will work, will work in the classroom*". Despite the separation of the locations of the pedagogy in university and school he still has faith in the effectiveness of the models that I teach him in his classroom context. He did not reveal the same level of trust in his mentor's approach to his teacher education; "*I felt that it was done to me. I received lots of stuff and I've been the one who has had to modify myself to the point where I've started to accommodate"*. Adam felt that he had to adapt to what was being '*done to'* him in school, adapt to the strong transmission orientation modelled by his mentor. For Adam, the first stages of learning to teach in a secondary mathematics classroom were situated in a culture characterised by transmission teaching, absorbed in activities that lead to an instrumental understanding of mathematics. He knew that adapting to this context "*takes away a piece of resilience"*. His capacity to apply what I have modelled in his classroom would be more difficult if his aptitude for recovering from disturbances has been dented, regardless of whether he had trust in the model's effectiveness.

Anna also understood the difficulties associated with implementing more connected pedagogical models; *"I think [mathematics teachers] all aspire to what you are saying but it just… it feels like a pipe dream. There are other pressures that you can't possibly plan for… this is reality".* The reality of learning to teach is situated in her secondary school classroom, where she knows that learning to teach in a connected manner is more complicated than resorting to the transmission model that her mentor adopted. In Anna's words: "*teachers* *tend to fall into the easy category and teaching connected is not easy"*. Anna perceives that teachers aspire to teach so that relational understanding is fostered, but that they cannot within the existing culture of the secondary school mathematics classroom. This suggests that the teachers that she works with believe in the principles of a more connected classroom, but that their actions in response to "*pressures that you can't possibly plan for"* results in transmission teaching. In this case, the disturbance caused by my presence in school is reduced by new information, the approach that I represent has merit, but is incompatible with the context of Anna's classroom. As with Adam's experience, this reduces the significance of my teaching when situated in the secondary mathematics classroom.

These difficulties were echoed by all of the participants in this study, perhaps with the exception of Ben. Rachel knew that developing relational understanding takes patience and time initially, but was constrained by the school context because "*you've got to get so much done in the certain amount of time*". Similarly, Sam reported the pressures of marking and report writing so that "*planning maybe takes a back seat"*. Luke was concerned about whether "*I can prove that they've made progress*" in the absence of notes and teacher feedback in the pupils books, echoing Anita's concerns about teachers in her department drowning under the demands of providing copious feedback in books in the manner that the school wanted. Yet the narrative in Ben's story consistently returns to his perception of how his pupils "*make sense of the maths*" and how they learn when "*they are able to see something"*. Marking, time constraints, extensive feedback and limited time to plan existed in Ben's school too, but were not cited as reasons for reverting to transmission teaching. He had "*absorbed everything*" from the university sessions because he could imagine how this model would support his pupils' learning, he was able to situate the experience based models within the culture and activity of his school.

John Seely Brown's (1989) links between the activity and enculturation of learning are as apparent in the experiences of my students as they are in that of the mathematics learning that he studied. His claim that "*much common educational practice is the victim of an inadequate epistemology*" (Brown et al, 2008, p. 42) has parallels with the experience of my students. They are victims of inadequate pedagogy modelled through the behaviour of the secondary mathematics communities of practitioners and the replicated culture and practice that characterise these communities. Transmission teaching serves as an inadequate pedagogy, but an unambiguous one, because it is the prevalent model seen by my students. Ambiguity is created in my students' pedagogical beliefs because of the separation of the locations of the university mathematics pedagogy and the prevailing model in schools. But whilst the transmission model dominates the school culture, my students' learning is situated within the activity and culture that signifies transmission teaching. Unless the more connected university model can be situated within the school culture, the activity associated with a connectionist orientation cannot be readily learned by my students.

Adam offered a solution to this dilemma. I offered him an occasional bridge between the usually unconnected worlds of university and school. Adam thought that there could and should have been more of that, more opportunity to situate learning models that develop a deeper relational understanding by bridging the two communities, by letting the two cultures merge in collective activity.

8.3 Incompatible Pedagogy

Transmission teacher orientation does not exclusively define the culture of learning in secondary school mathematics departments, but it does prevail. Askew (2002) popularised the framework of mathematics teacher orientations that permeate this study a century after Dewey described the following in his *Pedagogic Creed*:

> The teacher is not in the school to impose certain ideas or to form certain habits in the child, but is there as a member of the community to select the influences which shall affect the child and to assist him in properly responding to these influences.
>
> (1897, p. 9)

Yet a model that seems to "*impose certain ideas"* persists in the transmission teacher orientation. The connectionist orientated teacher selects the experiences that should stimulate mathematics learning, but does not leave the learner to discover mathematical concepts. Instead, the teacher assists the pupil to make connections by "*properly responding to"* the stimuli. My students are frequently teaching in a context that they perceive to be incompatible with a model of teaching that assists pupils in making connections, but one that is compatible with imposing a preconceived idea.

For most of the participants in this study, there are periods where the incompatibility of their university teacher education disturbs their teaching in school. Yet, they all agreed that I should continue to develop their mathematical pedagogical knowledge in the way that I do. The incompatibility of my pedagogy with the culture of school mathematics was a lived difficulty, but they felt that it was a necessary difficulty. Adam's "*complete trust and confidence"* in what I am teaching him gave my voice authenticity in the same way that Ben knew “*that’s what everyone should do*” because “*It just makes sense, build it up from basic principles”* when I taught him approaches to teaching at university. Rachel’s reasoning was similar, I have ‘*got to’* continue to teach her because of the depth of mathematical understanding that she has developed through her experiences at university sessions.

Like Ben and Adam, Sam was committed to developing a more connected classroom. She had faith in her commitment to ‘asking, not telling’, despite her mentor’s criticism, because she likes “*to think, in the long term, it’s going to stick with them a bit more*”. But for Luke and Rachel, the realisation of the effectiveness of pedagogical approaches came from experience in the classroom, their pupils’ failure to learn gave them a reason for seeking alternatives to a transmission orientation. “*Just put the numbers in, it’s not hard”* was ineffective, leaving Rachel open to models that build on pupils prior knowledge just as Luke rejected approaches modelled in school because *“I tried it with my bottom set and they didn’t like it”*. Each beginning teacher adopts an approach defined by their own beliefs about what is appropriate in the location of practice, which is derived from their own experiences, their beliefs about mathematics, about learning and about the situations in which the learning is positioned. This is illustrated by Ben’s desire to expose pupils to the richness of mathematics because he was not exposed to it himself, or Sam’s belief that knowledge should be built on known facts, or Adam’s faith in a connected model because he had seen how effectively I had used the approach in school or, ultimately, Luke and Rachel’s desire to improve learning when their early approaches fail. Each beginning teacher is navigating a complex landscape, characterised by uncertain classroom cultures and ambiguous pedagogies. Their actions, driven by their beliefs about what is appropriate in the contexts of the classrooms that they teach in, may not by synonymous with their underlying beliefs about mathematics education. Significantly, the PGCE classroom experience is characterised by being watched; not by me during the limited occasions that I bridge the world of university and school, but by their classroom teachers and mentors whose classrooms they borrow for a relatively short period of time.

Ruth's experience of adopting the 'peanut method' for adding fractions exemplifies a recurring theme for many of my students. Explicitly, *I believe in this* (a model that builds connections, conceptual development and reasoning alongside fluency) *but I do this* (teach in a manner that is characterised by transmission procedures and practising mathematics exercises in the pursuit of procedural fluency). Her belief that she will teach fractions in a reasoned manner was not altered by the experience of doing what her mentor asked her to do in 'delivering' the peanut method during her PGCE placement. She was operating in a complex landscape of learning to teach mathematics in a classroom that was situated in a culture of learning that is threatened by constraints of time and performativity. Was Ruth behaving with wisdom, teaching to please her driving instructor (to borrow Rachel's metaphor) so that she may qualify to teach and then *genuinely* learn to teach once she is away from the scrutiny of the mentor? Ruth's beliefs about pedagogy were incompatible with what her mentor was asking her to do, so she acted in a manner that was compatible with pleasing the instructor, so that she becomes licenced to develop her own pedagogy once qualified.

I did not observe any of Ruth's 'peanut method' lessons. I can only guess how the dialogue around her evaluation and feedback would have played out given how explicitly the university and school pedagogical worlds would clash in this case. No doubt, all parties would have been superficially respectful of each other's view, whilst implicitly or explicitly 'taking a pop' once the two worlds separate again. Thus, disturbance in the location of practice is reduced. Ruth's teaching is not only situated in the culture of learning within the community that she practices, but is situated in a culture of examination by the members of the community who uphold the practices and beliefs that characterise the community.

Ruth claimed that the peanut method incident increased her resolve to teach in a more connected manner, just as Sam did when her attempts to link sequences to number theory were criticised. At the end of their PGCE course, they both claimed that the incompatibility of their beliefs and the situated pedagogy of the schools had increased their desire to give their pupils opportunities to gain a deep mathematical understanding. It may be that the worlds of university and school teacher education do not need to be bridged for these students because their beliefs about teaching are, so they claim, more certain.

Yet other student teachers are less clear about the compatibility of their beliefs about pedagogy and their actions in the classroom. Resorting to faster, easier transmission teaching is a very real threat. Perhaps it is these student teachers who suffer most when they are positioned between divorcing parents. Subsequently, the days when the divorcing parents are getting along well have more significance for these student teachers.

Alternatively, perhaps I am providing a false justification for my influence in their evolving teaching practice. The seed of a connected classroom has been sown during university sessions and it may be that the student teachers' evaluations of their pupils' learning is sufficient to provide them with greater pedagogical certainty: they want to learn what works from experience.

An email from a student teacher early in the PGCE course suggests that the days when the divorcing parents get along, when we plan and teach collaboratively in one of the partnership schools, are significant:

> Although we have been over most of these concepts in University, it is very easy to forget once you are in school and dealing with other priorities and issues. Having this refresher session has made me stop and think and re-evaluate my teaching at just the right time. (Peter)

Like Adam, in the *Divorcing Parents* episode, Peter thinks that there could and should be more of these days where university and school teacher educators collaborate. On this day at least, the location of practice has also become the location of praxis. Peter was able to reflect on teaching episodes designed to model research informed practice, using a theoretical lens for interpretation with me, alongside a practical lens for interpretation with the school-based mentors. "*Just the right time"* is the point at which Peter is starting to realise the complexity of learning to teach mathematics. He had met research informed learning models in his induction period before he started teaching, but until these models have been situated in the location of his practice he is not able to "*stop and think and re-evaluate"* his teaching.

8.4 The situation of practice and the location of praxis

I do not expect my students to understand what signifies classrooms built on the connectionist orientation, unless they can understand features of classrooms that do not. Most report a dominant transmission orientation in their own secondary school experience, which is reproduced during many of their early PGCE observations in schools. For some, learning to teach mathematics becomes an exercise in reproducing the practices observed in school, with the outcome that early habits become instinctive practice. Time, difficulty and 'being watched' are cited as constraints on their practice, leading to the *"awful but easier"* transmission teacher orientation if their learning is situated within this constrained school culture. My teaching at the university is removed from this culture, separated by location as much as it is by pedagogy. In order to give my students a site for reflexivity in their practice I need to open up the possibility that they can critically reflect on their teaching and teach simultaneously. I need to give my students a site for reflection that leads to action, using Freire's notion of praxis; "*reflection and action upon the world in order to transform it"* (1970, p. 36). In this case, the part of the world that is transformed is my students' mathematics classroom. Freire's notion of praxis arose from his pedagogical developments in situations of Poverty in South America, leading to potentially emancipatory education and a more democratic model of learning than the one that existed in the 1960s. Freire's aim was to literally transform the world for South Americans of the lowest social and economic status, which does not necessarily translate to transforming the world of secondary school teacher education (Adams, Cochrane & Dunne, 2012). An absence of praxis for beginning teachers does not have the same consequences as for the subjects in Freire's work, but the principles of emancipation and agency still apply; in the absence of reflection and action in mathematics classroom the probability of transformative teacher education is limited.

Adam's "*bridge between the two worlds"* analogy suggests that it is possible for the worlds of school and university to connect. The day that allowed Peter to "*stop and think and re-evaluate*" was characterised by complementing pedagogical messages because the divorcing parents shared a belief and understanding about the effectiveness of certain aspects of pedagogy. There is, as Adam said, room for more days like that. But immersion days in schools should not form the exclusive model of my teacher education. My students do need to step away from the school environment in order to reflect on their teaching and to learn about research informed practice away from the school culture. Unless they step away from the situated culture, the only stimuli for understanding practice are the traits that characterise that culture. My student teachers cannot take action in their teaching, action that they govern while they are being watched by teachers who tell them, explicitly or implicitly how to teach. The main issue that separates my teaching from their school practice is their inability to respond reflexively to what they are learning because of the culture and activity that surrounds their situated learning in school.

The second issue relating to situated cognition, which poses a threat to my students' reflexivity, is that part of university-based teacher education that is perceived as wholly separate from what they do in school. When I ask my students to talk to me about their university teacher education, they directly reflect on aspects that are separate from their mathematics education sessions. The application of what they learn with me is immediately apparent, whereas aspects of whole school professional learning and research informed seminars relating to assignments are not immediately perceived as useful in what they are learning to do. They do not instinctively identify their mathematics education sessions as university teacher education until I direct them to. This was apparent in my interviews with all of the participants in the study as much as it is in end of course evaluation meetings with students. Their criticisms of these sessions suggest that an experience based model is absent. Paradoxically, some have commented that they are taught that transmission classroom orientations are less effective in lectures that are characterised by transmission. My teaching is located alongside the more separated world of general teacher education even though the pedagogy that I adopt is not situated within the culture of learning that they criticise. For some students, like Adam, Sam and Ben, my teaching has immediate authenticity. Yet for others, like Luke, Rachel and Anna, there is scepticism and doubt about how the mathematics pedagogy that they are taught can be enacted in school. It is possible that some of the authenticity, perceived as missing in general teacher education lectures, reduces the plausibility of what I am teaching them. For the sceptics, my teaching is situated in a culture that is removed from practice.

The bridge between the two worlds offers a route for channelling teacher education into school. It also offers an opportunity to step out of school, to stand and watch the school for a while, so that when student teachers return to school they might be able to act or think differently about teaching and learning than if they were exclusively immersed in the world of school.

Chapter 9 Critical Reflection on Findings

The previous chapter captures my findings from the four narrative episodes described in this study. The findings are summarised in the following themes:

- The disturbances experienced by my students as they shift their learning focus from their behaviour as teachers towards noticing the impact that their behaviour has on their pupils' learning; (Section 8.1)
- The influence of the culture of the school on my students' professional learning which is situated in classrooms borrowed for discrete periods during ITE and the resulting tensions that my students perceive; (Section 8.2)
- My students' perceptions of the incompatibility of the pedagogy situated in university mathematics education classrooms and the pedagogy modelled in school classrooms; (Section 8.3)
- The location of professional learning that either facilitates or stifles critical reflection on practice and the resulting actions stimulated by critical reflection. (Section 8.4)

In each of these themes, the student and beginning teachers describe tensions that exist in their perceptions of their early professional learning, influenced by university-based and school-based teacher educators. Tensions are apparent in their accounts of their beliefs about mathematics education and their justifications of how their beliefs influence their actions in the secondary mathematics classroom. These accounts address my research questions by capturing and synthesising my students' perceptions of learning to teach mathematics. This chapter moves towards my conclusion by critically reflecting on the wider implications of my small scale analysis of six teachers and the disturbances that their narratives have exposed.

The themes listed above are characterised by disturbances and discontinuities that are influenced by global, national and local concerns, each with a direct impact on ITE students. This study was introduced on the premise that recent policies of the 2010 coalition government are characterised by hasty political intervention into the structure of ITE in England. Policy influences that impact on mathematics teacher education and secondary mathematics education are my immediate concern, but are not necessarily the concern of my students. I suspect that many of my students are more concerned about whether their pupils will cooperate or whether their pupils will remember when to subtract their simultaneous equations. Likewise, national education policies influence my role within ITE but for my students, national curricula and Teachers' Standards are an unquestionable feature of their roles, but most do not pause to ask why the curriculum is structured in the way that it is because they are more concerned about how they will present the prescribed content to the pupils in their classrooms. However, local concerns are more immediately disturbing for me *and* my students. My students and I share concerns about the context and culture of the schools that they teach in, we share concerns about how their pupils make sense of the concepts that my students teach and we share concerns about how the university-based and school-based educators guide the development of their professional knowledge and practice. In essence, we share concerns about what is happening to them and the resulting consequences for their pupils.

9.1 Professional Knowledge

In setting the context for this study I described political influences on the landscape of learning to teach using Furlong's suggestion that research informed practice located in universities is squeezed out to be replaced by 'what works' practice located in school and culturally reproduced between teacher and teacher from school to school (2013). Ball's account of a culture of performativity describes the context of accountability and judgement that teachers are immersed in within their daily practices (2013) and the consequences of this culture upon student teachers' perceptions of 'what matters' in teaching mathematics (2006). Consequently, student teachers perceive university led mathematics teacher education as immediately useful in the classroom if it fits within the 'what works' notion of professional knowledge, because what is disseminated works without necessarily rationalising the research that informs the university model or the theory that underpins its potential effectiveness.

I do not propose that tension between knowledge situated in university-based and school-based teacher education is a relatively new phenomenon, because it is not. The emergence of the PGCE in the last sixty years has been characterised by disturbances between the perspectives of schools and universities in ITE. Stenhouse (1975) actively promoted the notion that the teacher's classroom was a research laboratory through which teachers develop professional expertise by integrating research and reflective practice; the classroom as the site of active research by the teaching professional and thus having agency in their own developing professionalism. The notion that reflection and practice combine in research informed activity in the classroom was developed further through the possibility that the classroom is the location of praxis (Freire, 1970; Grundy, 1987). In promoting classroom enquiry, Stenhouse (1967; 1975) justified the place of theoretical knowledge in the early stages of teacher education using a seed metaphor. The seed is sown in the early stages of teacher education, so that the plant of research informed and theoretically interpreted practice can grow once the teacher needs the theory. Stenhouse recognised that the immediate concerns of students in ITE differ from teachers with more experience of how their pupils learn. Without the opportunity to continually engage with research informed professional enquiry, once qualified, beginning teachers have limited opportunity for the seed to grow. Returning to school from university "*other pressures take over*" (Anna) even though teachers might believe in the principles behind research informed practice at university, there is no site to allow this to develop once in a school that is characterised by focus on nationally directed innovations and examination performance in a climate of performativity (Ball, 2003). Each school leaders' interpretation of government policy influences what signifies valuable and necessary practices, in a culture that is built on visible measures of performance within structured systems of monitoring and accountability. If research informed practice is not framed by policy makers as valuable or 'what matters' (Ball, 2013) then each student, who believes in developing a more connected pedagogy, has his or her sense of agency restricted by the school culture.

International comparisons identify strengths to systems of teacher professional knowledge that value on-going professional development using research to inform classroom enquiry (Darling-Hammond, 2010), but England is not identified as one of these systems. Zeichner

(2003) found characteristics of transformative teacher education, many of which are present in ITE, but are reported as absent in continuing professional development once my students have qualified. At university, students engage in research informed practice within a community of practitioners, where the focus of the enquiry is centred on their professional knowledge in relation to their pupils' learning. Professional knowledge that is centred on the replication of knowledge of what works from school to school and from classroom to classroom is not characteristic of professional knowledge that emancipates teachers' professional activity so that their actions can be driven by informed reflection and enquiry. There is an absence of sustained community enquiry informed by research, theory and systematic reflection on the learning that takes place in the teacher's classroom. As a result, the professional knowledge that is kindled in the university mathematics classroom during ITE is not sustained in future professional development in schools. This is an outcome of this study as much as it is the work of others (such as Askew, 1998: Zeichner, 2003) because the characteristics of transformative teacher education are frequently absent at the site of my students mathematics teacher education.

9.2 Disturbance

As mentioned previously, my presence in school disturbs the equilibrium of a mathematics department because I embody a representation of professional knowledge that differs from the new professionalism in schools. People respect what I do in ITE teaching, but do not accept that what I do has direct relevance to the pursuit of teacher knowledge that works in their classrooms because it is replicated from 'what worked' in another teacher's classroom. I have used Skemp's interpretation of Piaget's notion of disturbance to account for learning from conflicts that are exposed within a pupil's mathematical conceptual development. This idea is echoed in Festinger's theory of cognitive dissonance (1957), used to characterise the disturbance associated with learning that is not readily assimilated into existing concepts. Similarly, the model of professional knowledge that I represent when I am in school potentially disturbs the symbolic order of the secondary school mathematics department. To accept the validity of the knowledge that I represent many teachers' would need to accommodate new information into their existing schema for 'how my pupils learn to solve equations' or any other aspect of mathematics education under discussion when I am in school. Festinger (1957) identified the cognitive dissonance that this disturbance signifies and isolated conditions under which the new information is more likely to be disregarded, thus returning the individual to cognitive cognisance or returning to equilibrium in Piagetian terms. In many cases identified by Festinger, new information is dismissed more readily, if the firmly held beliefs of the learner have been acquired through struggle or difficulty. For many secondary school teachers, establishing purposeful classroom environments characterised by co-operative pupils and successful completion of tasks is reached through resilience and tenacity. The beliefs on which their classroom culture has been founded cannot easily be altered when the teachers' achievements in their classrooms have been won through personal toil. So, when the knowledge that I represent when I am in school causes cognitive dissonance, this disturbance can be readily alleviated by calling upon alternative information that reduces the significance of what I represent. Thus "*Sally is good at what she does*", at university, but what she does will not work in this context and the type of professional knowledge fostered in university teacher education is readily disregarded.

As discussed earlier, the professional knowledge that I embody is abjected by teachers in school (Oliver, 2002), because the existence of alternative conceptions of professional knowledge disturbs the beliefs of the teachers in the community, so that by abjecting my version of professional knowledge, the symbolic order of the mathematics teachers' community is maintained. This is more than taking a model that I suggest and rejecting its effectiveness through conscious consideration of the possibility of pupils learning from the model; abjection of the knowledge that I represent means that it is invalidated without question because any other possible model disturbs the symbolic order of the mathematics community and the beliefs that characterise the actions within that community in a manner that is inconceivable. This results in polite mistrust between the teacher educators that ITE students meet; university and school led models of classroom activity remain situated in the sites where they originate, so that ITE students remain caught between the two worlds articulated in this study.

As discussed earlier, the new National Curriculum in England (DfE, 2013) integrates principles from regions such as Singapore and Shanghai because these regions are judged to have more effective systems of mathematics education than England due to success in PISA test. This may raise the possibility that the research informed, theoretically interpreted version of university teacher knowledge is compatible with the 'what works' professional knowledge disseminated through the school-based 'maths hubs'. Aside from the obvious detail that this national curriculum is statutory for state maintained schools in England, it is possible that framing this knowledge as valuable because it works in Shanghai and Singapore, may reduce the resistance to the cognitive dissonance experienced by teachers when this knowledge is theoretically framed by university teacher educators.

However, the suggestion that to disseminate a Singapore style text book throughout English schools fails to acknowledge the fundamental social, cultural, historical, political and demographic differences between education systems in Singapore and England. Teacher education in England is situated in a context of social distain for mathematics (Williams, 2008) which results in social acceptance of failure to master mathematics. Generations of pupils in Singapore and Shanghai are growing up in families where basic principles of numeracy and literacy are firmly established before they start school at age six or seven. The mathematics that the pupils take home is recognisable to parents, because enduring learning models are renewed for subsequent generations. These are only a few of many differences that exist in mathematics education in England and higher performing countries, but serve to illustrate that transformation of mathematics education in this country is an enormous undertaking that cannot simply be achieved by replicating South East Asian text books or lesson structures in English schools. Mathematics education for this generation's PGCE graduates is the product of the cultural renewal of their experiences in relation to how they were taught in school, how they are taught at university and the contexts and cultures in which their teaching will be situated (Brown & McNamara, 2011). The beginning teachers' beliefs about what it means to learn or be taught mathematics is the product of the culture and context in which their learning and their pupils learning is situated (Lave, 1996: Brown, J. S., 1988) and the culture and contexts that exist are a product of the host of cultural, contextual, social and political influences (Wenger, 2009) that distinguish the English mathematics classroom and differ greatly from those in Shanghai and Singapore.

It is political influence that creates the greatest incompatibility between education systems in Shanghai and England. As Gill describes "*Neoliberalism is increasingly understood as constructing individuals as entrepreneurial actors who are rational, calculating and self-regulating.*" (2008, p. 441). Thus the principles that reproduce and transform professional knowledge in the regions that the DfE aspire to replicate are impossible under the current neoliberal movement in England. Government policy dictates 'what matters' in education and so has a direct influence on how teachers' professional knowledge is framed and valued. Confidence in teachers' professional integrity has been eroded in England for decades (Ball, 2013) because of the prevalence of external measures of accountability seen, for example, through the political rhetoric of 'failing schools'. Globally, the most successful education regions, as measured through attainment in PISA, are characterised by ongoing development of teachers' professional knowledge, developed locally, and not solely external measures of accountability (Darling-Hammond, 2010). In the culture currently dominating English schools, student mathematics teachers may be left to work pedagogy out for themselves when situated in a context of individual accountability within externally governed measures of performativity.

9.3 Individual Teacher Knowledge

The ideological shift needed to implement a mastery curriculum cannot be achieved by transferring lesson structures and textbooks from other countries. Affecting the beliefs and practices of an education system towards a mastery model requires a colossal transformation in the beliefs and practices that characterise secondary mathematics education, set within the social and political ideology that limits practitioners' capacity to transform.

When my PGCE students first attend my classes, they are regularly disturbed by the problems that I give them. They will frequently leap to an analytical solution to a problem, showing me their mathematical fluency with symbolic representations by presenting elegant, precise, direct algebraic proofs of the problem set. Their satisfaction is justified as they sit back and relax; they have reached a level of fluency that exposes mastery in at least one aspect of the curriculum that I am educating them to teach. But they are then asked to justify their solutions in words, they are asked to reason why their solution should convince me that a problem has been solved. They are asked to convince me in any way other than algebraic, using diagrams, graphs, pictures, actions, stories and so the list of questions goes on. Their initial solution demonstrates to me that they have sound subject knowledge, which is vitally important and so, is usually established as a pre-requisite of starting the course. My task is then to develop my students pedagogical mathematical knowledge (Shulman, 1987) so that they can move freely between representations of mathematics, so that they can understand the nature of variation in the pursuit of generalisations, so that they can reason beyond algebraic representations and so that, above all, they are sensitised to the many ways that the pupils in their classes may make sense of mathematics. Their resistance to this cognitive dissonance is palpable and I share my interpretation of this disturbance with them; I license them to feel uncomfortable because I understand what this aspect of teacher education feels like.

Some students embrace multiple representations of mathematical concepts more readily than others. They seem to understand the theoretical model of translation between enactive, iconic and symbolic representations (Bruner, 2006a) and they try to find out which representation would advantage the pupils that they teach. Early attempts at fostering enactive models tend to leap to symbolic forms prematurely, or they may foist enactive and iconic representations onto their pupils when some have a sound symbolic understanding already. Others remain indifferent until they have experienced the absence of symbolic understanding in their classrooms. None of this is straightforward for my students, but they are adopting an enquiring approach within their ITE year that allows them to reflect on their actions and the actions of their pupils as they develop their teacher knowledge.

Others emphatically resist. Students who have reached mastery of symbolic representations of mathematics may not be aware of how they reached their level of mastery on a metacognitive level, but they have firmly held beliefs that their conception of mathematical fluency is sufficient to equip them as teachers. Their pupils can learn to reach a similar level of mastery by listening intently to their explanations, following their carefully modelled examples, practising their sensibly selected exercises and responding affirmatively to their accurately corrected bookwork. For some students, this approach is validated by 'what works' in the contexts where their teaching is situated.

Most of my students are adept at telling the stories of mathematical concepts through symbolic representations. However, the pedagogical model that I teach them requires them to expand their symbolic understanding, or possibly undo it, so that they can make sense of enactive and iconic representations. Figure 9.1 represents the direction of learning within these representations, with many secondary school pupils in one direction and many of my PGCE students in the other.

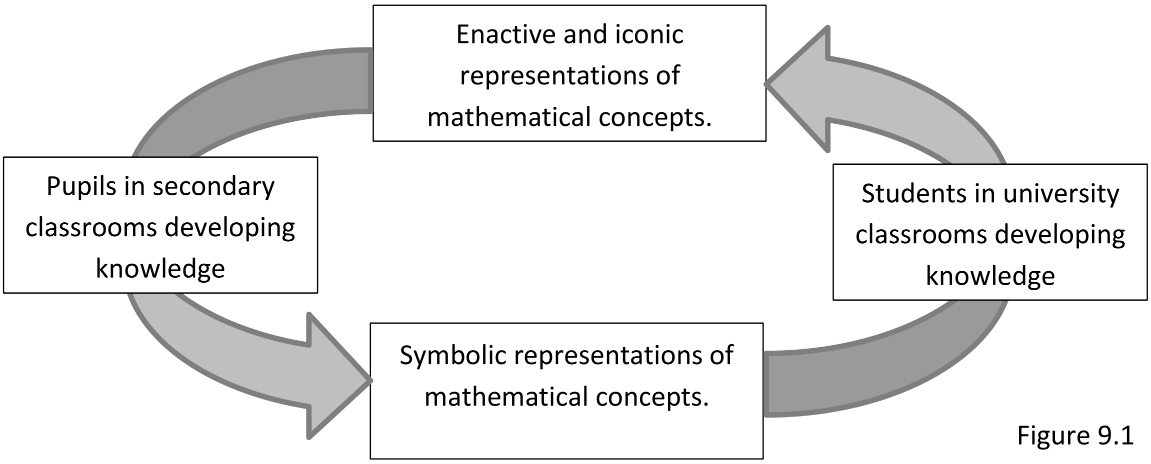

Figure 9.1

I know that some pupils in secondary schools, those termed as higher attainers, are probably adept at manipulating symbolic representations of mathematics. However, many pupils are not demonstrating a fluent comprehension of mathematical concepts and as studies such as the ICCAMS project illustrate (Hodgen et al., 2009a, 2009b), England has made little or no progress in improving this in the last thirty years. I think that it is likely that many of the pupils in my students' classrooms would benefit from iconic and enactive representations of mathematics.

This exposes another discontinuity in my students' professional knowledge. Secondary school mathematics teachers, fluent in symbolic representations of mathematics are required to undo their belief that symbolic fluency embodies mathematical fluency and become open to alternative representations as they expand their knowledge of what mathematics is and how it is learned.

In some cases, my students are open to the possibility that a deep understanding of mathematics requires multiple representations, but do not have a sense of agency in the classrooms that they are operating in; resulting in the feeling that their ideas are prematurely quashed before attempts at professional enquiry have been tried.

In each of these illustrations, professional knowledge influences different learning trajectories for each student (Wenger, 2013). The individual student's professional knowledge influences their openness to approaches to teaching in ways that only their dispositions, structured from their beliefs and experience can bear. The professional knowledge of the teachers who educate them in school influences the students' agency in planning and teaching their own lessons, in acquiring knowledge of the context in which they teach and the culture of the school that they teach in. The human interactions in the classroom influence the decisions that student teachers make in action on a minute by minute basis. Student teachers know that they are being assessed, which influences their actions in the classroom so that the structure of ITE policies influence their articulation of their professional knowledge. My professional knowledge influences their conceptions of mathematics and mathematical teacher knowledge, as do the experiences that I expose my students to in school and at the university. Each interaction, each influence, changes the learning trajectory for the student teacher.

Learning to teach mathematics is fundamentally complicated because of the many influences identified above and many more that are pertinent to the individual teacher and the contexts where they operate. Not all of the influences are antagonistic and not all experiences of conflict have negative consequences, being a necessary part of learning, but some do and it is those aspects of teacher education that cause harm to my students and to the pupils that they teach that are my concern. Ultimately, it is those influences that disqualify students from learning by informed professional enquiry, coupled with an absence of continued stimulation for professional learning once qualified that concern me.

Chapter 10 Conclusion

This study set out to understand student teachers' perceptions of learning to teach mathematics and capture their beliefs about mathematics education within the context of university-led and school-led teacher education. This has been achieved through my ethnography by framing my analysis of their narratives within my understanding of the phenomenon of learning to teach mathematics. This study has exposed tensions that exist in my students' perceptions of learning to teach and has articulated the nature of these tensions in relation to my students' beliefs about mathematics education, my students' sense of agency in their emerging professional roles, the nature of situated cognition, the dominant school culture prevalent at the time of this study and the nature of professional knowledge framed in school and at university.

Dewey published *Experience and Education* (1938) to clarify his position on education having seen how the progressive education movement of the early 1900s in the USA had become obscured in the name of his democratic education principles. He identified flaws in progressive education that had become the opposite of what he intended (Dewey, 1938; Mason & Johnston-Wilder, 2004). Progressive education was never intended to be discovery learning that reduced the role of the teacher to a passive bystander in the learning process. Askew and Swan distinguished connectionist teacher orientations from discoverist to make the same distinction as Dewey (1938). The teacher plays a vital role in the connectionist classroom by choosing the influences that the learner needs in order to make connections within and between mathematical concepts. The learners are not left to their own devices to 'rediscover' thousands of years of mathematical constructs, but nor is knowledge transmitted to passive recipients of learning as seen in Dewey's criticisms of traditional education and Askew and Swan's criticism of transmission teaching.

In 2006 Kirschner, Sweller and Clark published a paper demonstrating the failure of current education ideology integrating constructivist learning principles because of the absence of direct teacher instruction and guidance. The much-cited paper elicited responses from researchers (Hmelo- Silver, Duncan & Chinn, 2006) who were quick to illustrate that Kirschner and his colleagues had misrepresented the principles of constructivism by aligning its principles alongside discovery learning instead of those principles that complement the connectionist teacher orientation. It seems that this is a simple mistake to make, because I have been challenged by numerous mathematics teachers who have suggested that I am misguided because I want pupils to discover mathematics for themselves. This is neither possible nor true. I want pupils' mathematics education to be stimulated so that they make connections within and between mathematics concepts, so that they build understanding through representations that relate to their experience and so that they realise the challenges and pleasures that learning mathematics affords. To do this, I think the pupils need to be guided by a teacher, one that is adept at asking them questions (appropriate questions at appropriate times) using stimuli that affect the pupils' experience, one that explains concepts with clarity and a teacher who believes in the potential of all pupils to learn mathematics given the conditions that make learning imaginable for them. This is what I believe, but I know perfectly well how difficult this is to achieve. Like Stenhouse (1967), I believe in sowing a seed of research and theoretically informed enquiry during initial teacher education, but I am unsure whether the present dominant model of professional knowledge in England allows the seed to ever germinate. The data apparent within this study expose only a few of the misunderstandings apparent between me and my students or between me and the teachers who educate my students in school. The data is riddled with discontinuities that serve to exemplify the potential disturbances that my students are exposed to at the start of their teacher education.

Teacher education in universities and schools are separated ideologically as much as they are by location, yet everyone I know who is involved in mathematics education enters the profession because they want pupils to learn mathematics well, taught by good teachers. I share the same goal as the teachers in schools, but my actions are situated away from the location of responsibility for the pupils' learning. I have no direct contact with the pupils in school because my responsibility to them is, at best, implied by my responsibility to their teachers who I educate. The teachers who educate my students in school have explicit and direct responsibility for the mathematical attainment of their pupils, most urgently measured by their attainment in examinations, but signified by other measures such as their pupils' willingness to study mathematics beyond the compulsory age of 16 and their attitude and aptitude to mathematics once they leave school. These issues concern me, but I do not shoulder the burden of responsibility for these issues. I am not found wanting if fewer pupils choose to study mathematics post 16 or if fewer pupils make expected levels of progress in secondary school mathematics classrooms.

Regulations within ITE, and my universities reaction to these regulations, ensure that there are other performativity measures to raise the possibility that I could be found wanting, such as ensuring that I recruit people who develop into beginning teachers who are judged to be at least ‘good’ with respect to assessment criteria, who remain on the course until they gain QTS and who are retained as successful teachers once qualified. I am held accountable for all of these things. I can sympathise with the teachers that Ball studied who related the ‘terrors of performativity’ found in their professional roles (2003). Not least because I was a classroom teacher at the time of Ball’s research, but also because those influences bleed into every aspect of education. Maintaining the integrity of professional beliefs is difficult to do in a context where professional identity is skewed by locally applied interpretations of external measures of performativity, which creates influences on teachers’ and teacher educators’ actions. This, ultimately, alters classrooms and the professionals who teach within them.

I am not yet able to offer a ‘better’ model of mathematics teacher education, but my thesis does provide some compelling evidence that the system that currently exists is at best inconsistent and at worst divisive. Furthermore, my study provides evidence that the nature of professional knowledge that dominates my students’ perceptions of mathematics education in schools stifles the development of teachers’ pedagogical mathematical knowledge. Presently, there is little space for reflexivity and limited agency for beginning teachers, which is compounded by the absence of research informed, theoretically interpreted practice in school. Neoliberal culture is not conducive to professional enquiry because knowledge that is incompatible with ‘what matters’, as determined by propagators of education policy, disturbs the current dominance of examination driven school culture. Derived from this evidence, my thesis has exposed beginning mathematics teachers' perceptions of their experiences and their beliefs about mathematics education, emphasising the disturbances that are apparent in the two sites of their teacher education. Consequently, there are several themes that have emerged that merit further research, which are described in the following questions.

- Is it possible to find a site for research informed initial teacher education, where discontinuities within the worlds of school and university are smoothed and the contribution of all parties is valued? It is neither desirable nor possible to eradicate difference, because each principle that informs professional knowledge should be understood for what it is as much as what it is not. Difference is healthy, when each party respects and trusts the contribution of the other.

- Is it possible to design programmes of initial and continuing teacher education that embed the principles of professional enquiry? Short term professional development opportunities do little to transform practice whereby a site for sustained collaborative enquiry is missing.

- Is it possible to design mathematics teacher education programmes that embed different representations of mathematics through structures that allow more pupils to learn to master mathematics? Cultural reproduction of mathematical knowledge is healthy when the outcome results in most people within that culture gaining mastery.

At the heart of the discontinuities exposed in this study there exists a group of beginning teachers whose voices have been heard throughout. These students are learning and all learning involves an element of disturbance. However, it is those disturbances that erode trust between participants: that disqualify students from learning by informed professional enquiry or stifle their agency in school that should concern educators and policy makers. It is fitting to end with a comment from Adam as he reflected on one of his lessons in which he felt that the trust between the teacher and the pupils was significant. This is not an attempt to find a happy ending in all of these disturbances, but is intended as a reminder that while teacher education, and possibly education is in a state of flux, individual teachers are trying to make mathematics learnable and individual pupils are trying to learn.

> My personality is starting to come through now. That little moment was quite a big moment for me, when you get that kind of reaction and then you are *with* the kids then and they get it. I felt that they thought 'I'm with them… we're all going along together' and I was with them and trying to encourage and support them. I was able to use the approach that *I* chose and I felt confident to do that. It's on another level. You're not just worrying about classroom control, it's a much richer experience. That sense of trust in the classroom. (Adam)

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